THE SOLAR INGRESS (*SANKRĀNTI*) ACCORDING TO THE *MAKARANDASĀRIṆĪ* AND OTHER INDIAN ASTRONOMICAL TEXTS

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Abstract: In the present paper we analyze the procedure for the computation of the sidereal solar ingress according to the popular Indian astronomical table, the *Makarandasāriṇī*. The results are compared with those obtained from the basic treatise *Sūryasiddhānta*, from the *Vākya* and the *Gaṇakānanda*, and also from those based on modern computations.

We have also discussed the varying durations of the solar months and the solar ingress to the twenty-seven *nakṣatras* (zodiacal asterisms). A number of illustrative examples are also provided.

Keywords *sankrānti*, *nakṣatra*, *Makarandasāriṇī* (*MKS*), *saurapakṣa*, *Gaṇakānanda* (*GNK*), *sauramāna*, *cāndramāna*, *adhikamāsa*

1 INTRODUCTION

Saṅkrānti is the instant when the Sun enters a *rāśi* (sidereal zodiac sign). In Indian astronomy a sidereal solar year commences when the Sun enters *Meṣa*, the sidereal sign for Aries. Currently this occurs around 14–15 April, but due to the precision of equinox this date shifts by one day in about 72 years.

In Indian society, the *Meṣa saṅkrānti* plays an important socio-religious role. In the Hindu calendar, religious festivals are celebrated either according to the solar calendar (*sauramāna*) or the lunar calendar (*cāndramāna*). For example, in regions like Tamil Nadu, Kerala, West Bengal and Dakshina Kannda in Karnataka the solar calendar is adopted. On the other hand in most of the other parts of India like Karnataka, Maharashtra and Andhra the lunar calendar is followed.

The solar months (*māsas*) are generally named after the Sun's entry into *rāśis* (sidereal signs) such as *Meṣa* (Aries), *Vṛṣabha* (Taurus) etc. But more popularly, the names of the solar months are the same as those of the lunar months viz., *Caitra*, *Vaiśākha* etc.

Most of the Hindu festivals are based on the luni-solar (or lunar) calendar. For example *Kṛṣṇajanmāṣṭamī* and *Sri Rāmanavamī* etc. are based on the lunar calendar. On the other hand, the festival of *Makara Saṅkrānti* (*Pongal* festival) and Tamil New Year's day (*Sauramānayugādi*) are based on the solar calendar. The famous Kerala festival *Tiruoṇam* is observed annually in the solar month of *Siṁha* when the Moon occupies the *Śravaṇa nakṣatra* (lunar mansion).

In the following sections we discuss the tables for the determination of *Saṅkrānti* given in the *Makarandasāriṇī* (*MKS*). The procedure for this determination as also to find the durations of the successive solar months are discussed mathematically. Examples are provided to illustrate these procedures.

The solar ingress into the 27 *nakṣatras* is also discussed from the corresponding tables of the *MKS*. In fact, the durations of the Sun's occupation of these *nakṣatras* are called *Mahānakṣatras*. The farmers reckon the seasons by these *Mahānakṣatras*. For example, the wet season in Karnataka, due to the South-West Monsoon, ranges over about ten *Mahānakṣatras*, from *Rohiṇī* to *Hasta*.

The generation of the related tables is analyzed mathematically. Results obtained using the *Makarandasāriṇī* are compared with those derived from other texts such as the *Vākya*, the *Gaṇakānanda* and the *Sūryasiddhānta* (*SS*). It should be noted that while the *MKS* and the *GNK* are based on the *SS*, the *Vākya* system is based on the *Āryabhaṭīya* of Āryabhaṭa I (b. C.E. 476).

2 THE *SŪRYASIDDHĀNTA*, THE *MAKARANDASĀRIṆĪ* AND THE *GAṆAKĀNANDA*

The currently-popular *Sūryasiddhānta* is said to date from the ninth or tenth century C.E. Prior to that the above name referred to one of the five systems provided in the *Pañcasiddhāntikā* of Varāhamihira. In order to distinguish it from the *Sūryasiddhānta*, Varāhamihira's version is generally called the *Saurasiddhānta*.

Traditional Indian astronomical almanacs, called *pañcāṅgas*, are compiled annually, based on popular treatises like the *Sūryasiddhānta*, the *Āryabhaṭīya* (C.E. 499), Brahmagupta's *Brāhmasphu-ţasiddhānta*, (C.E. 628), and Gaṇeśa Daivajña's *Grahalāghava* (C.E. 1520). The *pancāṅgas* generally are computed using various astronomical tables (*sāriṇīs*).

In this paper we consider mainly the *Makarandasāriṇī* (*MKS*) and *Gaṇakānanda* (*GNK*) astronomical tables, both of which belong to the *Saurapakṣa* (school) following the *Sūryasiddhānta*. The *Vākya* system, popular in southern India, is a set of simple and meaningful Sanskrit sentences that are pneumonics of letter numerals that represent the true positions of the heavenly bodies. In this paper, we compare the results of the *MKS* and the *GNK* with those of the *Vākya* system.

2.1 The *Gaṇakānanda*

The *Gaṇakānanda* is a popular text in the Andhra and Karnataka regions among almanac makers. It is a *karaṇa* (manual) text in which the computations of heavenly bodies are calculated from mid-noon. Apart from the textual part, it also contains the astronomical tables. The text is based on the *Sūryasiddhānta* and authored by Sūryacārya, the son of Bālāditya who came from the Andhra region. The ephocal date of the text is 16 March 1447. Incidentally, there was a solar eclipse on that day. The author incorporated the word-numeral system (*bhūtasaṅkhyā*) in explaining the procedure and methods.

2.2 The *Makarandasāriṇī* (*MKS*)

The *Makarandasāriṇī* is a tantra text based on the *Sūryasiddhānta*. The *Makarandasāriṇī* was composed in C.E. 1478 by Makarandācārya, the son of the almanac-maker Ānanda. Makarandācārya made quite a few innovations in the procedures to calculate planetary positions and eclipses. Commentaries on the *Makarandasāriṇī* by Dai-vajña Divākara, called the *Makarandavivaraṇa* and the *Udāharaṇa* by Daivajña Viśvanātha, are available. The highlight of the text is that the author has reduced four stages of correction (*phalasaṁskāras*) to only three by combining the half *manda* and the full *manda* corrections together to obtain the true planet.

2.3 The *Vākya* Tables

The *Vākyakaraṇa* is an astronomical text composed by a Vararuci in the early thirteenth century. It is the most popularly-used text to construct almanacs in the southern parts of India, with the commentary by Sundararāja. The *Vākya* tables belong to the *āryapakṣa*, based on the parameters and procedures of Āryabhaṭīya and mainly on the works of Bhāskara I's *Mahābhāskarīya*. The *Vākyas* are given in the form of *Kaṭapayādi* notations, a system of letter numerals. This text consists of six astronomical chapters which are very useful to the *pañcāṅga*-makers to find the positions of the heavenly bodies in order to perform rituals. The five chapters are:

(i) The true positions of the Sun, the Moon and *rāhu* (the Moon's ascending node);
(ii) The true positions of the five planets;
(iii) Problems involving time, position and direction;
(iv) Eclipses;
(v) Heliacal rising and setting; and
(vi) Parallel aspects (*Mahāpātas*).

3 TABLES GIVEN IN THE *MKS* FOR FINDING *SAṄKRĀNTI*

In this Section, Table 1 (below) gives the *saṅkrāntikṣepaka* (additive) for *śaka* years with an interval of 24 years. The *saṅkrāntikṣepaka* is expressed in *vāra* (^{vā}, or week day), *ghaṭi* (^{gh}) and *vighaṭi* (^{vg}), where

1 day = 60 *ghaṭis*
1 *ghaṭi* = 24 minutes
1 *ghaṭi* = 60 *vighaṭis*
1 *vighaṭis* = 24 seconds

Table 1 can be generated by adding $2^{vā}$ 12^{gh} 36^{vg} to the previous entry, as shown below in Table 2.

In the printed copies of Viśvanātha's commentary on the *MKS*, the table of *saṅkrāntikṣepaka* is given from *śaka* years 1568 to 1808. In Table 1, the values are given up to *śaka* year 2000 by extending the table by adding $2^{vā}$ 12^{gh}

Table 1: *Saṅkrāntikṣepaka* for *śaka* years.

śaka year	1544	1568	1592	1616	1640	1664	1688	1712	1236	1760	1784	1808	1832	1856	1880	1904	1928	1952	1976	2000
Christian Year (CE.)	1622	1646	1670	1694	1718	1742	1766	1790	1814	1838	1862	1886	1910	1934	1958	1982	2006	2030	2054	2078
vāra	5	0	3	5	0	2	4	0	2	4	6	1	4	6	1	3	6	1	3	5
ghaṭi	41	53	6	19	31	44	56	9	22	34	47	59	12	25	37	50	2	15	28	40
vighaṭi	17	53	29	5	41	17	53	29	5	41	17	53	29	5	41	17	53	29	5	8

36^{vg}. The additive constant of $2^{vā}\ 12^{gh}\ 36^{vg}$ is obtained by considering the duration of 24 solar years.

Thus we have

$$365^{d}.2587565 \times 24 = 8766^{d}.210156$$
$$= 8766^{d}\ 12^{gh}\ 36^{vg}.5616$$

In the text it is taken as $2^{vā}\ 12^{gh}\ 36^{vg}$ (by removing the integral multiples of 7 from the integer part).

Table 3 gives the *vāra* (week day), *ghaṭi* and *vighaṭi* of the *saṅkrāntikṣepaka* for the balance years (*śeṣavarṣa*) from 1 to 24. This table is obtained by adding *vārādikṣepaka* 1|15|31|30 for each entry correspondingly from year 2 to 24 and taking approximate integer in the *vighaṭi* position. However, the accumulation of error is reduced for large number of years.

Table 4 gives *saṅkrāntikṣepaka* (additives) of the 12 *rāśis* (signs) from *Meṣato Mīna* (Sidereal Aries to Pisces). Adding these 12 *rāśi saṅkrāntikṣepaka* (additive) values to *abdapa* of the given *śaka year* for a given place one can obtain the *vāra* (week day), *ghaṭi* and *vighaṭi* of the Sun's entry into the different *rāśis*.

4 PROCEDURE FOR OBTAINING THE *SAṄKRĀNTI* FOR A GIVEN *ŚĀLIVĀHANA ŚAKA* YEAR ACCORDING TO THE *MKS*

The following working procedure is adopted in *MKS* for finding *saṅkrānti* (Sun's entry into 12 *rāśis*).

(1) Find the nearest *Śālivahana* (śā) *śaka* (*śeṣa varṣa*) year for the given *śā.śaka* year using Table 1 and obtain the difference between the given *śā.śaka* year and the nearest *śaka* year from the table. The difference is called *śeṣa-varṣa.*
(2) Find *vārādi (vāra, ghaṭi and vighaṭi)* for the nearest *śaka* year using Table 1 and for *śeṣa varṣa* (the balance years) using Table 3.

Table 2: An example illustrating how Table 1 was generated.

	Śaka year	*vāra*	*ghaṭi*	*vghaṭi*
	1568	0	53	53
Adding	24	2	12	36
	1592	3	06	29
Adding	24	2	12	36
	1616	5	19	05

(3) Add *vārādi* obtained for the nearest *śaka* year and *śeṣa varṣa* (balance years).
(4) Add the *deśāntara* correction (the correction in time made for the longitude of the place) to the above sum. The result is called the *abdapa* for the given place in the given *śā.śaka* year. Thus, the *abdapa* is the constant for the given solar year, for the given place, representing the beginning *vāra* (weekday), *ghaṭi* and *vighaṭi*.
(5) Add the *saṅkrāntikṣepaka* given in the table 4 for each *rāśi* from *Meṣa* to *Mīna* to the above obtained *abdapa*. The result gives the *vāra* (day), *ghaṭi* and *vighaṭi* of each *saṅkrānti* (Sun's entry into 12 *rāśis*) correspondingly to from *Meṣa* to *Mīna.* The above procedure is illustrated in the following examples.

Table 3: *Saṅkrāntikṣepaka* for the balance years

śeṣavarṣa	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
vāra	1	2	3	5	6	0	1	3	4	5	6	1	2	3	4	6	0	1	2	4	5	6	0	2
ghaṭi	15	31	46	2	17	33	48	4	19	35	50	6	21	37	52	8	23	39	54	10	26	41	57	12
vighaṭi	31	3	35	6	38	9	41	12	44	15	47	18	50	21	53	24	56	27	58	30	2	33	5	36

Table 4: *Saṅkrāntikṣepaka* for *Meṣādirāśi* (Zodiac signs)

rāśi	*Meṣa*	*Vṛṣabha*	*Mithuna*	*Karkaṭaka*	*Simha*	*Kanyā*	*Tulā*	*Vṛścika*	*Dhanus*	*Makara*	*Kumbha*	*Mīna*
vāra	0	2	6	3	6	2	4	6	1	2	4	5
ghaṭi	0	57	23	0	30	30	55	48	17	36	3	53
vighaṭi	0	1	1	51	4	29	48	31	11	4	15	21

Table 5: The day, *ghaṭi* and *vighaṭi* of the Sun's entry into the 12 *rāśis*.

Sl. No.	*Rāśis*	*abdapa+kṣepaka*	day, *ghaṭi*and*vighaṭi*
1	*Meṣa*	0|30|45 + 00|00|00	0|30|45 i.e. Sun enters into *Meṣa*on Saturday at 30^{gh} 44^{vig}
2	*Vṛṣabha*	0|30|45+ 02|57|01	3|27|46 i.e. Sun enters into *Vṛṣabha* on Tuesday at 27^{gh} 45^{vig}
3	*Mithuna*	0|30|45+ 06|23|01	6|53|46 i.e. Sun enters into *Mithuna* on Friday at 53^{gh} 45^{vig}
4	*Karkaṭaka*	0|30|45+ 03|00|51	3|31|36 i.e. Sun enters into *Karkaṭaka*on Tuesday at 31^{gh}35^{vig}
5	*Simha*	0|30|45+ 06|30|04	7|00|49 i.e. Sun enters into *Simha*on Saturday at 00^{gh} 48^{vig}
6	*Kanyā*	0|30|45+ 02|30|29	3|01|14 i.e. Sun enters into *Kanyā*on Tuesday at 01^{gh}13^{vig}
7	*Tulā*	0|30|45+ 04|55|48	5|26|33 i.e. Sun enters into *Tulā*on Thursday at 26^{gh}32^{vig}
8	*Vṛścika*	0|30|45+ 06|48|31	7|19|16 i.e. Sun enters into *Vṛścika*on Saturday at 19^{gh}15^{vig}
9	*Dhanus*	0|30|45+ 01|17|11	1|47|56 i.e. Sun enters into *Dhanus*on Sunday at 47^{gh}55^{vig}
10	*Makara*	0|30|45+ 02|36|04	3|06|49 i.e. Sun enters into *Makara*on Tuesday at 06^{gh} 48^{vig}
11	*Kumbha*	0|30|45+ 04|03|15	4|34|60 i.e. Sun enters into *Kumbha* on Wednesday at 33^{gh}59^{vig}
12	*Mīna*	0|30|45+ 05|53|21	6|24|06 i.e. Sun enters into *Mīna* on Friday at 24^{gh}05^{vig}

(5) Add the *saṅkrāntikṣepaka* given in the table 4 for each *rāśi* from *Meṣa* to *Mīna* to the above obtained *abdapa*. The result gives the *vāra* (day), *ghaṭi* and *vighaṭi* of each *saṅkrānti* (Sun's entry into 12 *rāśis*) correspondingly from *Meṣa* to *Mīna.* The above procedure is illustrated in the following examples.

4.1 Example Number 1

This example is taken from Viśvanātha's commentary on the *MKS*.

Given *śaka* year = 1551 (1629 C.E.). The nearest *śā.śaka* year from Table 1 is 1544; the balance years (*śeṣavarṣa*) = 1551 – 1544 = 7; *vārādi* for the nearest *śaka* year 1544 using Table 1 is 5|41|17. *Vārādi* for the balance years (*śeṣavarṣa*) 7 using Table 3 is 1|48|41. Adding we get 7|29|58. Adding the *deśāntara* correction (for *Kāśī*) → 0|00|47 to the above we have *abdapa → 7|30|45. Thus*, the *abdapa* (for *Kāśī) for the given śaka* year 1551 is 7|30|45 ≡ 0|30|45 (removing multiple of 7). Now adding the *saṅkrāntikṣepaka* given in Table 4 for each *rāśi* from *Meṣa* to *Mīna* (sidereal Aries to Pisces) to the above obtained *abdapa* we get the day, *ghaṭi* and *vighaṭi* of the Sun's entry into 12 *rāśis*. This is shown in Table 5.

4.2 Example Number 2

Given *śaka* year = 1939 (C.E. 2017). The nearest *śā.śaka* year from Table 1 is 1928; the balance years (*śeṣavarṣa*) = 1939 – 1928 = 11; *vārādi* for the nearest *śaka* year 1928 using Table 1 is 6|02|53. *Vārādi* for the balance years (*śeṣavarṣa*) 11 using Table 3 is 6|50|47. Adding we get 12|53|40. Adding the *deśāntara* correction (for *Kāśī*) → 0|00|47 to the above we have *abdapa* → 12|54|27. Thus, the *abdapa* (for *Kāśī*) for the given *śaka* year 1939 is 12|54|27 ≡ 5|54|27 (removing multiple of 7). Now adding the *saṅkrāntikṣepaka* given in the Table 4 for each *rāśi* from *Meṣa* to *Mīna* to the above obtained *abdapa* we get day, *ghaṭi* and *vighaṭi* of the Sun's entry into 12 *rāśis*. This also is shown in Table 5.

5 DURATION OF SOLAR MONTHS

5.1 Durations of Solar Months

We have derived the formula for finding the Sun's entry (*saṅkrānti)* into different *rāśis* by considering the mean duration of a solar month, the mean daily motion of the Sun and the equation of the centre (*mandaphala*) of the Sun according to the *Sūryasiddhānta*.

Duration of *Saṅkrānti* = Mean duration –

$$\left[\frac{13.5}{2\pi \times SDM}\left\{\sin MK_{end} - \sin MK_{beg}\right\}\right]$$

where MK_{end} and MK_{beg} are the *mandakendra* (anomaly from the apogee) of the Sun at the end and at the beginning of *Saṅkrānti* respectively and *SDM* is the Sun's daily motion.

The interval between two successive *rāśisankrāntis* is defined as a solar month (*sauramāsa*). For example, the interval between the *Meṣasankrānti* and the *Vṛṣabhasankrānti* is the length of the *Meṣamāsa.* However the durations of the twelve solar months are not uniform. The mean length of a solar month according to the *SS* can be obtained from the number of civil days (*sāvanadinas*) in a *Mahāyuga* of 432 × 10^4 sidereal solar years.

5.2 *Sankrānti* According to the GNK

Civil days according to *SS* are 1577917828. We have the sidereal (*nirayaṇa*) solar year = $1{,}57{,}79{,}17{,}828/432 \times 10^4$ = 365.2587565 days. The maximum equation of the centre (*mandaphala*) i.e., when the *mandakendra* is 90° is 130.32′.

The equation of the centre = $\frac{p}{2\pi}$ sin90° = 2° 10′ 19.2″ hence *p* = 13° 38′ 50″. By taking *p* as 13° 40′ we get the *mandaphala* as 2° 10′ 30.42″ i.e. the *GNK* has taken the variable *paridhi* (periphery) from 14° to 13° 40′ as the *MK* (*mandakendra*) varies from 0° to 90°.

Lunar months in a *mahāyuga* (*MY*) = 53433336. ∴ One lunar month = 29.53058795 days and hence one lunar year = 354.367055 days. Solar year – Lunar year = 10.8917015 days. The difference in *tithis* = $11^{tit}\ 3^{gh}\ 53.4^{vig}$, where a *tithi* is $1/30^{th}$ of a lunar month. A mean solar month = 365.2587565/12 = 30.43822971 days = 30|26|17.63 days.

The *Saṅkrāntikṣepakas* for the different *rāśis* according to the *GNK* are given in Table 6.

5.3 Duration of the Solar Month According to the *GNK*

Since the text (*GNK*) is based on the *Sūryasiddhānta*, the formula for the Sun's entry (solar ingress) into different *rāśis* is derived by considering the mean duration of the solar month etc. according to the *SS*.

The duration of *Saṅkrānti* = Mean duration –

$$\frac{1}{2\pi \times SDM}\left[P(N+1)\sin(MKR)_1 - P(N)\sin(MKR_2)\right]$$

where $P(N + 1)$ and $P(N)$ are the peripheries at the end of even and odd quadrants the MKR_1 and MKR_2 are the *mandakendras* (anomalies) at the end and the beginning of *saṅkrāntis* respectively and *SDM* is Sun's daily motion.

The solar month in which the Sun is at apogee (*mandocca*) is of maximum duration while the solar month in which the Sun is at perigee is of minimum duration. Currently, the Sun is at apogee around 4 July, and at perigee around 2 or 3 January. Correspondingly, the durations of the sidereal solar months of *Mithuna* and *Dhanus* (sidereal Gemini and Sagittarius) are respectively maximum and minimum.

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Table 6: The *Saṅkrāntikṣepakas* for *Meṣādirāśis* (according to the *GNK*).

1	*Meṣa*	11|5|31
2	*Vṛṣabha*	2|55|32
3	*Mithuna*	6|19|41
4	*Karkaṭaka*	2|56|22
5	*Simha*	6|24|34
6	*Kanyā*	2|26|44
7	*Tulā*	4|54|6
8	*Vṛścika*	6|48|13
9	*Dhanus*	1|18|37
10	*Makara*	2|39|30
11	*Kumbha*	4|6|46
12	*Mīna*	5|55|10

The lengths of the twelve sidereal solar months according to the *SS*, *MKS*, *Vākya* and the *GNK* and modern computations, are compared in Table 7.

Note the differences among the values between *the sāriṇīs* and the modern ones:

(1) The *mandaparidhi* of the Sun is taken as constant and equal to 13.5° by the Āryabhaṭan school. The *Vākya* system belongs to this school.
(2) In the *saurapakṣa* based on the *Sūryasiddhāntha*, the Sun's *mandaparidhi* is taken as variable, between 13° 40′ and 14°, based on the *manda* anomaly (*mandakendra*).
(3) Since there were no calculators and computers in ancient times, computations of the trigonometric function *jyā*(Rsine) were based on approximate tabular values. Here too, the *jyā* tables in the *Āryabhaṭīya* and the *Sūryasiddhānta* are provided at intervals of 3° 45′.

Many other texts give the *jyā* tables at even bigger intervals of 10° or 15°.

Table 7: Durations of the solar months.

Sl. No.	*Rāśis*	*Sūryasiddhāntha* (*SS*)	*Makarandasāriṇī* (*MKS*)	*Vākya*	*Gaṇakānanda* (*GNK*)	Modern
1	*Meṣa*	30|58|3.76	30|57|1	30|57|1	30|56|52	30|52|8.94
2	*Vṛṣabha*	31|22|47.24	31|26|37	31|23|4	31|25|36	31|16|27.95
3	*Mithuna*	31|33|49.46	31|37|50	31|33|54	31|35|54	31|27|12.2
4	*Karkaṭaka*	31|26|15.36	31|29|13	31|26|37	31|30|10	31|21|0.94
5	*Simha*	31|03|51.59	31|00|25	31|03|10	31|06|01	30|59|48.95
6	*Kanyā*	30|31|59.16	30|25|19	30|29|50	30|30|08	30|29|54.06
7	*Tulā*	29|57|9.76	29|52|43	29|55|33	29|53|07	29|59|33.62
8	*Vṛścika*	29|28|0.98	29|28|40	29|29|30	29|25|56	29|36|38.36
9	*Dhanus*	29|14|15.79	29|18|53	29|18|40	29|16|0	29|26|48.37
10	*Makara*	29|23|43.51	29|27|11	29|25|57	29|27|56	29|32|27.2
11	*Kumbha*	29|50|34.26	29|50|06	29|40|24	29|46|38	29|52|12.35
12	*Mīna*	30|25|0.77	30|56|20	30|22|44	30|22|47	30|21|12.96

6 TABLES GIVEN IN THE *MKS* TO FIND THE SUN'S ENTRY INTO THE *NAKṢATRAS* (LUNAR MANSIONS)

6.1 Obtaining the Sun's Entry into the Twenty-seven *Nakṣatras* According to the *MKS*

The Sun's entry into the different *nakṣatras* can be obtained by using the following procedure:

(1) Obtain the *abdapa* for the given *śaka* year using Tables 1 and 3, as explained in the case of the *Mesa saṅkranti* (in Section 4). The *abdapa* marks the beginning of the sidereal solar year in weekdays (*vāra*), *ghaṭis and vighaṭis.*
(2) Add the *abdapa* to the *kṣepakas* of the *nakṣatras* given in Table 8. The result gives the weekday, *ghaṭi and vighaṭi* of Sun's entry into different *nakṣatras* respectively.

6.1.1 Example 1

This example is taken from Viśvanātha's commentary on the *MKS*.

Given *śaka* year = 1551 (C.E. 1629). The *abdapa* (for *Kāśī*) for the given *śaka* year 1551 is 7|30|44 ≡ 0|30|44 (removing multiples of 7). Now adding the *nakṣatrakṣepaka* given in Table 8 for each *nakṣatra* from *Aśvinī* to *Revatī* to the above *abdapa* we get the day, *ghaṭi* and *vighaṭi* of the Sun's entry into the twenty-seven *nakṣatras*. These are listed in Table 9.

6.1.2 Example 2

Given *śaka* year = 1939 (C.E. 2017). The *abdapa* (for *Kāśī*) for the given *śaka* year 1939 is 12|54|27 ≡ 5|54|27. Now, the Sun's entry into the twenty-seven *nakṣatras* for the *śaka* year 1939 (C.E. 2017) is obtained by adding *nakṣatrakṣepaka* given in Table 8, as shown in Table 10.

Note that the time duration (in integral number of days) between the Sun's entry from one *rāśi* to another *rāśi* is about 30 days, and the same from one *nakṣatra* to another *nakṣatra* is 14 days.

This explains why the entries to some successive *nakṣatras* fall on the same weekday.

Table 8: *Sankrāntikṣepaka* for 27 *nakṣatras.*

Aśvinī	*Bharaṇī*	*Kṛttikā*	*Rohinī*	*Mṛgaśira*	*Ārdrā*	*Punarvasu*	*Puṣya*	*Āśleṣa*	*Makhā*	*Pubba*	*Uttara*	*Hasta*	*Cītrā*	*Svātī*	*Viśākhā*	*Anurādhā*	*Jyeṣṭhā*	*Mūlā*	*Pūrvāṣāḍha*	*Uttarāṣāḍha*	*Śravaṇa*	*Dhaniṣṭhā*	*Śatabhiṣaj*	*Pūrvabhādra*	*Uttarābhādra*	*Revatī*
0	6	6	6	6	6	6	6	6	6	6	6	5	5	4	3	3	2	1	0	6	5	4	3	2	2	1
0	41	30	24	23	24	29	32	30	30	19	5	43	11	36	56	5	12	17	19	19	26	30	40	51	11	42
0	34	0	35	36	57	38	39	40	4	41	42	43	44	45	46	47	48	11	50	51	0	0	0	0	0	0

Table 9: The Sun's entry into the different *nakṣatras* for *śaka* year 1551.

Sl. No.	*Nakṣatras*	*abdapa+kṣepaka*	Weekday, *ghaṭi* and *vighaṭi* of the Sun's entry into *Nakṣatras*
1	*Aśvinī*	0|30|45 + 00|00|00	0|30|45i.e. Saturday at 30^{gh}44^{vig}
2	*Bharaṇī*	0|30|45 + 06|41|34	0|12|19i.e. Saturday at 12^{gh}18^{vig}
3	*Kṛttikā*	0|30|45 + 06|30|00	0|00|45i.e.Saturday at 00^{gh}44^{vig}
4	*Rohinī*	0|30|45 + 06|24|35	6|55|20i.e. Friday at 55^{gh}19^{vig}
5	*Mṛgaśira*	0|30|45 + 06|23|36	6|54|25i.e. Friday at 54^{gh} 24^{vig}
6	*Ārdrā*	0|30|45 + 06|24|37	6|55|22i.e.Friday at 55^{gh} 21^{vig}
7	*Punarvasu*	0|30|45 + 06|29|38	0|00|23i.e. Saturday at 00^{gh} 22^{vig}
8	*Puṣya*	0|30|45 + 06|32|39	7|03|24 i.e. Saturday at 3^{gh} 23^{vig}
9	*Āśleṣā*	0|30|45 + 06|32|40	7|03|25 i.e. Saturday at 3^{gh} 24^{vig}
10	*Makhā(Maghā)*	0|30|45 + 06|30|04	7|00|49 i.e. Saturday at 0^{gh} 48^{vig}
11	*P.Phālguṇī*	0|30|45 + 06|19|41	6|50|26 i.e. Friday at 50^{gh} 25^{vig}
12	*U. Phālguṇī*	0|30|45 + 06|05|42	6|36|27 i.e. Friday at 36^{gh} 26^{vig}
13	*Hasta*	0|30|45 + 05|43|43	6|14|28 i.e. Friday at 14^{gh} 27^{vig}
14	*Cītrā*	0|30|45 + 05|11|44	5|42|29 i.e. Thursday at 42^{gh} 28^{vig}
15	*Svātī*	0|30|45 + 04|36|45	5|07|30 i.e. Thursday at 7^{gh} 29^{vig}
16	*Viśākhā*	0|30|45 + 03|56|46	4|27|31 i.e. Wednesday at 27^{gh} 30^{vig}
17	*Anurādhā*	0|30|45 + 03|05|47	3|36|32 i.e. Tuesday at 36^{gh} 31^{vig}
18	*Jyeṣṭhā*	0|30|45 +02|12|48	2|43|33 i.e. Monday at 43^{gh} 32^{vig}
19	*Mūlā*	0|30|45 + 01|17|11	1|47|56 i.e. Sunday at 47^{gh} 55^{vig}
20	*Pūrvāṣāḍhā*	0|30|45 + 00|19|50	0|50|35 i.e. Saturday at 50^{gh} 34^{vig}
21	*Uttarāṣāḍha*	0|30|45 + 06|19|51	6|50|36 i.e. Friday at 50^{gh} 35^{vig}
22	*Śravaṇa*	0|30|45 + 05|26|00	5|56|45 i.e. Thursday at 56^{gh} 44^{vig}
23	*Dhaniṣṭhā*	0|30|45 + 04|30|00	5|00|45 i.e. Thursday at 0^{gh} 44^{vig}
24	*Śatabhiṣaj*	0|30|45 + 03|40|00	4|10|45 i.e. Wednesday at 10^{gh} 44^{vig}
25	*Pūrvābhādrā*	0|30|45 + 02|51|00	3|21|45 i.e. Tuesday at 21^{gh} 44^{vig}
26	*Uttarābhādrā*	0|30|45 + 02|11|00	2|41|45 i.e. Monday at 41^{gh} 44^{vig}
27	*Revatī*	0|30|45 + 01||42|00	2|12|45 i.e. Monday at 12^{gh} 44^{vig}

Table 10: The Sun's entry into the different *nakṣatras* for *śaka* year 1939.

Sl. No.	*Nakṣatras*	*abdapa+kṣepaka*	Weekday, *ghaṭi* and *vighaṭi* of the Sun's entry into *Nakṣatras*
1	*Aśvinī*	5|54|27 + 00|00|00	5|54|27 i.e. Thursday at 54gh 27vig
2	*Bharaṇī*	5|54|27 + 06|41|34	5|36|01 i.e. Thursday at41gh 34vig
3	*Kṛttikā*	5|54|27 + 06|30|00	5|24|27 i.e.Thursday at 24gh 27vig
4	*Rohinī*	5|54|27 + 06|24|35	5|19|02 i.e. Thursday at 19gh 02vig
5	*Mṛgaśira*	5|54|27 + 06|23|36	5|18|03 i.e.Thursday at 18gh 3vig
6	*Ārdrā*	5|54|27 + 06|24|37	5|19|04 i.e. Thursday at 19gh 4vig
7	*Punarvasu*	5|54|27 + 06|29|38	5|24|05 i.e. Thursday at 24gh 5vig
8	*Puṣya*	5|54|27 + 06|32|39	5|27|06 i.e. Thursday at 27gh 6vig
9	*Āśleṣa*	5|54|27 + 06|32|40	5|27|07 i.e. Thursday at 27gh 7vig
10	*Makhā*	5|54|27 + 06|30|04	5|24|31 i.e. Thursday at 24gh 31vig
11	*Pubba*	5|54|27 + 06|19|41	5|14|08 i.e. Thursday at 14gh 8vig
12	*Uttara*	5|54|27 + 06|05|42	5|00|09 i.e. Thursday at 0gh 9vig
13	*Hasta*	5|54|27 + 05|43|43	4|38|10 i.e. Wednesday 38gh 10vig
14	*Citrā*	5|54|27 + 05|11|44	4|06|11 i.e. Wednesday at 6gh 11vig
15	*Svātī*	5|54|27 + 04|36|45	3|31|12 i.e. Tuesday at 31gh 12vig
16	*Viśākhā*	5|54|27 + 03|56|46	5|51|13 i.e. Thursday at 51gh 13vig
17	*Anurādhā*	5|54|27 + 03|05|47	2|00|14 i.e. Monday at 0gh 14vig
18	*Jyeṣṭhā*	5|54|27 + 02|12|48	1|07|15 i.e. Sunday at 7gh 15vig
19	*Mūlā*	5|54|27 + 01|17|11	0|11|38 i.e. Saturday at 11gh 38vig
20	*Pūrvāṣāḍha*	5|54|27 + 00|19|50	6|14|17 i.e. Friday at 14gh 17vig
21	*Uttarāṣāḍha*	5|54|27 + 06|19|51	5|14|18 i.e. Thursday at 14gh 18vig
22	*Śravaṇa*	5|54|27 + 05|26|00	4|20|27 i.e. Wednesday at 20gh 27vig
23	*Dhaniṣṭhā*	5|54|27 + 04|30|00	3|24|27 i.e. Tuesday at 24gh 27vig
24	*Śatabhiṣaj*	5|54|27 + 03|40|00	2|34|27 i.e. Monday at 34gh 27vig
25	*Pūrvabhādra*	5|54|27 + 02|51|00	1|45|27 i.e. Sunday at 45gh 27vig
26	*Uttarābhādra*	5|54|27 + 02|11|00	1|05|27 i.e. Sunday at 5gh 27vig
27	*Revatī*	5|54|27 + 01|42|00	0|36|27 i.e. Saturday at 36gh 27vig

Table 11: Duration of the Sun's entry into different *nakṣatras*.

Sl. No.	*Nakṣatras*	*Makarandasāriṇī* (*MKS*)	*Vākya*	*Sūryasiddhāntha* (*SS*)
1	*Aśvinī*	13.69277	13.69011232	13.70138
2	*Bharaṇī*	13.80722	13.7963388	13.80175
3	*Kṛttikā*	13.90972	13.88810456	13.88827
4	*Rohinī*	13.98361	13.9604625	13.95755
5	*Mṛgaśira*	14.016944	14.00951178	14.0066
6	*Ārdrā*	14.083611	14.03260813	14.0328
7	*Punarvasu*	14.050277	14.02850644	14.02755
8	*Puṣya*	14.00027	13.99742781	13.99417
9	*Āśleṣa*	13.95666	13.94104772	13.93876
10	*Makhā*	13.82694	13.86240562	13.86398
11	*Pubba*	13.666694	13.76574113	13.77291
12	*Uttara*	13.63388	13.65626546	13.66915
13	*Hasta*	13.466944	13.53988047	13.55669
14	*Citrā*	13.416944	13.4228605	13.44018
15	*Svātī*	13.3336111	13.31151413	13.32495
16	*Viśākhā*	13.1502777	13.21184408	13.21707
17	*Anurādhā*	13.11694	13.12922358	13.12298
18	*Jyeṣṭhā*	13.073056	13.06810672	13.04916
19	*Mūlā*	13.044167	13.03178833	13.00148
20	*Pūrvāṣāḍha*	13.00028	13.02222634	12. 98678
21	*Uttarāṣāḍha*	13.1025	13.03993625	13.01275
22	*Śravaṇa*	13.06667	13.08396331	13.06879
23	*Dhaniṣṭhā*	13.166667	13.151934	13.14937
24	*Śatabhiṣaj*	13.183333	13.24018401	13.2483
25	*Pūrvabhādra*	13.333333	13.34395575	13.35906
26	*Uttarābhādra*	13.516667	13.45765488	13.47526
27	*Revatī*	12.3	13.57515183	13.59108

6.3 Derivation for the Durations of the Sun's Entry into Different *Nakṣatras*

The duration of Sun's entry into *nakṣatra* = the mean duration –

$$\left[\frac{13.5}{2\pi \times SDM}\left\{\sin MK_{end} - \sin MK_{beg}\right\}\right]$$

where MK_{end} and MK_{beg} are *mandakendras* of the Sun at the end and at the beginning of a *nakṣatra* respectively and *SDM* is Sun's daily motion. Since the mean sidereal solar year is 365.2587565 days, the mean duration of a *nakṣatra* = 365.2587565/27 = 13.52810209 = 13|31|41.17days.

Table 11 compares the reported duration of the Sun's entry into different *nakṣatras*, as listed in the *MKS, Vākya* and *SS*.

7 CONCLUDING REMARKS

In the preceding sections we have discussed in detail the procedures for determining the

(1) beginning of the sidereal solar year for any given *śālivāhanaśaka* year i.e. the *Meśasankrānti* of that year;
(2) weekdays, *ghatis* and *vighaṭis* of the Sun's entry into each *raśi*;
(3) durations of the 12 solar months;
(4) weekdays, *ghatis* and *vighaṭis* of Sun's entry into each of 27 *nakṣatra*; and
(5) duration of the Sun's stay in each *nakṣatra*.

These results, according to Makarandasāriṇī, are compared with those in the *Sūryasiddhānta*, *Vākya* and *Gaṇakānanda* and modern computations.

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