THE BURMESE *UḌHAYA* PROCEDURE: A SIMPLIFIED CALCULATION OF THE OBLIQUE ASCENSION OF THE SUN

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Abstract: The Burmese *uḍhaya* procedure represents a simplified calculation of the oblique ascension of the Sun that was used in Indian astronomy and astrology. The calculation has obvious connections with an Indian astronomical work, the *Pauliśasiddhânta*, and with works written by Brahmagupta. We investigate the Burrnese procedure mathematically to find out why it works.

Keywords: Burmese astronomy, India, oblique ascension, day length, approximation

1 INTRODUCTION

The oblique ascension is an important quantity in Indian and Southeast Asian astronomy/astrology (Gislén and Eade, 2019: 461). The oblique ascension gives the rising times of the different zodiacal signs at a specific location that in turn gives knowledge of the rising sign or the lagna used for computing horoscope data.

The oblique ascension Ω of the Sun is given by the formula

$$\Omega = \alpha - A \qquad (1)$$

where α is the right ascension of the Sun and A the ascensional difference. The right ascension is a function of the Sun's longitude and does not depend on the observer's location while the ascensional difference depends on both the solar longitude and the observer's location.

The right ascension can be computed from the solar longitude, λ, by

$$\tan \alpha = \cos \varepsilon \cdot \tan \lambda \qquad (2)$$

with ε being the obliquity of the ecliptic, in an Indian context taken as 24°. The ascensional difference A is the difference between the time of the rising of a body and six o'clock. For the Sun it would also be the increase of a half day of 6 hours (in angular measure equal to 90°) and can be calculated from

$$\sin A = \tan \phi \cdot \tan \delta \qquad (3)$$

For negative declinations this expression will be negative and the half day will be short-er than 6 hours.

ϕ is the geographical latitude of the observer's location, and δ, the declination of the Sun, is given by:

$$\sin \delta = \sin \varepsilon \cdot \sin \lambda \qquad (4)$$

For solar longitudes of multiples of 30°, we can write $\lambda = s \cdot 30°$, with s = 0, 1, 2, ... 11 being the zodiacal sign. The oblique ascension for the beginning of signs can then be written as Ω (ϕ, s) to express that it is only a function of the observer's location and the sign of the Sun in the zodiac.

Southeast Asian astronomy uses the time units *nadi* and *vinadi* of which there are 60 *nadis* in a day and night and 60 *vinadis* in each *nadi*, and thus 3600 *vinadis* in a day and night. The *vinadi* unit is quite handy as it is easy to convert from degrees to *vinadi* by a simple multiplication by 10 as 3600 *vinadi* corresponds to 360°. A *vinadi* is equal to 24 seconds of time.

For geographical latitude ϕ = 0, the equator, we can calculate the oblique ascensions Ω (0, s) once and for all. We will be interested in the differences created by the variation of solar longitude

$$\Delta (\phi, s+1) = \Omega (\phi, s + 1) - \Omega (\phi, s)$$

and for ϕ = 0 we can compute these differences expressed in vinadi units and rounded to whole numbers as shown in Table 1.

Table 1: Differences in oblique ascensions for geographical latitude zero.

Δ(0,1)	Δ(0,2)	Δ(0,3)	Δ(0,4)	Δ(0,5)	Δ(0,6)
278	299	323	323	299	278

We will only need the first six numbers as for any location we have the mirror symmetry

$$\Delta (\phi, s) = \Delta (\phi, s - 11) \qquad (6)$$

i.e. the last six numbers are the first numbers in reverse order. In order to remember the sequence you only need the first three numbers in Table 1. These numbers are standard in Indian and Southeast Asian astronomy and are given in the *Sûryasiddhânta* in units of respirations as 1670, 1795, 1935 respectively (Burgess, 2000). If these numbers are divided by 6, the number of respirations in a *vinadi* and then rounded, we reproduce the first three numbers of the Burmese series.

For locations different from the equator we can now calculate the ascensional differences $A(\phi, s)$ for signs s = 0, 1, 2, .., 11 using formulas (4) and (3). The differences in oblique ascension for the observer's latitude ϕ will then be:

$$\Delta(\phi, s+1) = \Delta(0, s) - (A(\phi, s+1) - A(\phi, s)) = \Delta(0, s) - \Delta A(\phi, s+1), \quad (7)$$

with $\Delta A(\phi, s+1) = A(\phi, s+1) - A(\phi, s)$.

2 THE APPROXIMATION

The *uḍhaya* procedure as communicated to me by private communication (Ko Ko Aung, pers. comm., 12 April 2014) gives a short-cut recipe for calculating the quantities for ΔA and for input only needs the height of the equinoctial shadow at the observer's location. One of the meanings of uḍhaya (उदय) in Hindu astronomy is the rising of a planet on the eastern horizon.

The *uḍhaya* procedure is as follows:

1) Start from the length of the equinoctial shadow for a gnomon with height 12 units.
2) Multiply this shadow length by 60. This means that effectively the gnomon height is 720 units.
3) Make three columns, in the first one multiply the shadow length by 10 and divide by 60. Keep the quotient rounded to the nearest integer. This will be $\Delta A(\phi, 1)$.
4) In the second column multiply the shadow length by 8 and divide by 60. Keep the quotient as before, this will be $\Delta A(\phi, 2)$.
5) In the third column multiply the shadow length by 3 and divide by 60. Keep the quotient, this will be $\Delta A(\phi, 3)$.

These three ΔA numbers are called *sar-atta*. My Burmese source says the multiplier for the third column is four but the internal coherence of the procedure indicates that this is in error for three as the multiplier, as is shown below. The Burmese numbers three and four are mirror images of each other (၃ and ၄) and sometimes are confused.

6) The local oblique ascension differences will then be

$$\Delta(\phi, 1) = \Delta(0, 1) - \Delta A(\phi, 1)$$
$$\Delta(\phi, 2) = \Delta(0, 2) - \Delta A(\phi, 2)$$
$$\Delta(\phi, 3) = \Delta(0, 3) - \Delta A(\phi, 3)$$
$$\Delta(\phi, 4) = \Delta(0, 4) + \Delta A(\phi, 4)$$
$$\Delta(\phi, 5) = \Delta(0, 5) + \Delta A(\phi, 2)$$
$$\Delta(\phi, 6) = \Delta(0, 6) + \Delta A(\phi, 1) \quad (8)$$

The last six oblique ascension differences are given by the mirror symmetry.

There are some obvious connections with India. In the *Pauliśasiddhânta* (Neugebauer and Pingree, 1970(I): 41; 1970(II): 29; Sastry,1993: 48) we find a similar prodedure with multipliers 20, 16 1/2, and 6 3/4 that are used to calculate twice the ascensional differences and which are very nearly twice the Burmese multipliers, 10, 8, and 3. Brahmagupta (Kaye, 1998: 80) gives 19, 16 1/4, and 6 for these Indian multipliers.

As an example we apply the procedure for Yangoon with latitude 16° 39′. The equinoctial shadow of a gnomon of 12 units is then $12 \cdot \tan\phi = 3.589 \approx 3{:}36$. We multiply by 60 to bring this value to 215. The steps in the procedure now give

$$\Delta A(1) = 215 \cdot 10 / 60 = 36 \quad \Delta A(2) = 215 \cdot 8 / 60 = 29. \quad \Delta A(3) = 215 \cdot 3 / 60 = 11 \quad (9)$$

If we then take this result and use the standard numbers from Table 1 together with the formula (8) we get:

$$\Delta(\phi, 1) = 278 - 36 = 242$$
$$\Delta(\phi, 2) = 299 - 29 = 270$$
$$\Delta(\phi, 3) = 323 - 11 = 312$$
$$\Delta(\phi, 4) = 323 + 11 = 334$$
$$\Delta(\phi, 5) = 299 + 29 = 328$$
$$\Delta(\phi, 6) = 278 + 36 = 314 \quad (10)$$

These are precisely the values you get by an exact calculation and are those given in astrological handbooks (Mauk, 1971)—see Figure 1. This figure shows the zodiac with the oblique ascension differences for Yan-

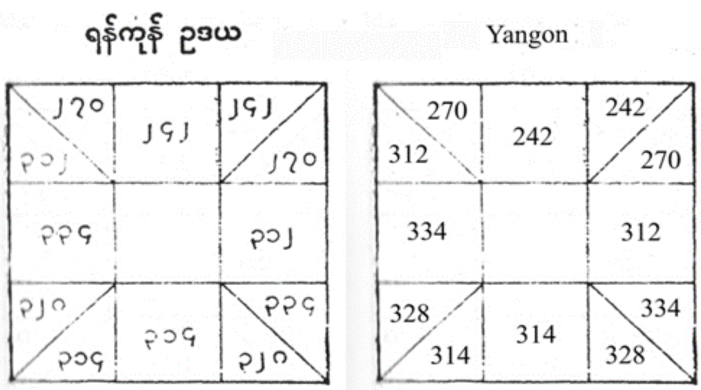

Figure 1. Yangon *zata* with oblique ascension differences (after Mauk, 1970).

goon. The diagram represents the zodiac with sign 0 at the middle top and the following signs anti-clockwise around the diagram.

The question is now why this simple procedure works.

3 ANALYSIS

We can easily compute the declinations of the Sun for signs 0, 1, 2, and 3 ($\lambda = 0°, 30°, 60°$, and 90°) using formula (4) with the Southeast Asian obliquity $\varepsilon = 24°$. The corresponding declinations are 0°, 11.734°, 20.625°, and 24°. The ascensional differences by formula (3) then are

$$\sin A(\phi, 1) = \tan \phi \cdot \tan(11.735°) = \tan \phi \cdot 0.2077$$
$$\sin A(\phi, 2) = \tan \phi \cdot \tan(20.625°) = \tan \phi \cdot 0.3764$$
$$\sin A(\phi, 3) = \tan \phi \cdot \tan(24°) = \tan \phi \cdot 0.4452 \quad (11)$$

The ascensional differences A for locations in Burma will be quite small angles and we can use the fact that the sine of a small angle is almost the same as the angle itself in radians. In fact for the northern limit of Myanmar with a geographical latitude of 28.5° the error of this approximation would be only about 1%. This is also the approximation used in the *Pauliśasiddhânta* (Neugebauer and Pingree, 1970(II): 29). We then get

$$A(\phi, 0) = 0$$
$$A(\phi, 1) = \tan \phi \cdot 0.2077$$
$$A(\phi, 2) = \tan \phi \cdot 0.3764$$
$$A(\phi, 3) = \tan \phi \cdot 0.4452 \quad (12)$$

Taking differences we have:

$$\Delta A(\phi, 1) = \tan \phi \cdot (0.2077 - 0) = \tan \phi \cdot 0.2077$$
$$\Delta A(\phi, 2) = \tan \phi \cdot (0.3764 - 0.2077) = \tan \phi \cdot 0.1687$$
$$\Delta A(\phi, 3) = \tan \phi \cdot (0.4452 - 0.3764) = \tan \phi \cdot 0.0688 \quad (13)$$

Here the ΔAs are still expressed in radians.

The length of the equinoctial shadow of a gnomon with length 12 is $12 \cdot \tan \phi$. The length E used in the algorithm above is sixty times this, thus $E = 60 \cdot 12 \cdot \tan \phi = 720 \cdot \tan \phi$ from which we get

$$\tan \phi = E / 720 \quad (14)$$

We also convert the angles in radians to vinadis by first multiplying by $180/\pi$ to express it in degrees and then by 10 to convert it to vinadis, in total by $1800/\pi$. Inserting these changes we get:

$$\Delta A(\phi, 1) = E \cdot 0.2077 / 720 \cdot 1800/\pi = E \cdot 0.1631$$
$$\Delta A(\phi, 2) = E \cdot 0.1687 / 720 \cdot 1800/\pi = E \cdot 0.1325$$
$$\Delta A(\phi, 3) = E \cdot 0.0688 / 720 \cdot 1800/\pi = E \cdot 0.0547 \quad (15)$$

The multiplier in step 3 of the *uḍhaya* procedure is 10/60 = 0.1667, the multiplier in step 4 is 8/60 = 0.1333, and the multiplier in step 5 is 3/60 = 0.0500. These numbers are quite close to the multipliers in equation (13) and show that the *uḍhaya* procedure is usable. The error in the two first multipliers is only respectively 2% and 1%. The largest error will be in the last multiplier, about 10%, causing an error of about 1 unit in the *vinadi*.

The cumulative sum of the *saratta* numbers, the *sarathawa*, will give the ascensional differences or the increase or decrease of the mean half day of 6 hour or 15 *nadis* or 900 *vinadis*. In the case of Yangoon as an example the cumulative *sarattas* will be 0, 36, 36 + 29 = 65, 36 + 29 + 11 = 76 and the half days of the first six signs 0, 1, 2, 3, 4, 5, 6 will be respectively 900, 936, 965, 976, 965, 936, 900 *vinadis*.

The *Pauliśasiddhânta* procedure is the same but deleting the multiplication and division by 60:

> Multiply the constants 20, 16½, and 6¾ by the equinoctial shadow. The results are the [double] ascensional differences in vināḍīs, first in the given order, then in the reverse order for the first six months and again the given and reverse orders for the second half of the ecliptic, the second six months. (Sastry,1993: 48).

The multipliers are doubled because what is meant here by ascenional differences are the total changes of day length.

In his explanation of the Cambodian calculation of daylength Faraut (1910: 196) gives just a circular diagram for the ascensional differences without any derivation (Figure 2). The differences in *vināḍīs* for the first three zodiacal signs: 40, 32, and 14 respectively are found in the outer rim of the diagram. Assuming the same approximation procedure as before and dividing these numbers by four, being equal to twelve times the equinoctial shadow, we get precisely 10, 8, and 3.5, very nearly half the Indian multipliers. The equinoctial shadow would correspond to a geographical latitude of 18.43° N in central Southeast Asia. It seems very probable that this was a common approximation procedure.

4 CONCLUSION

We have shown that this Burmese astronomical approximation is a quite efficient way of

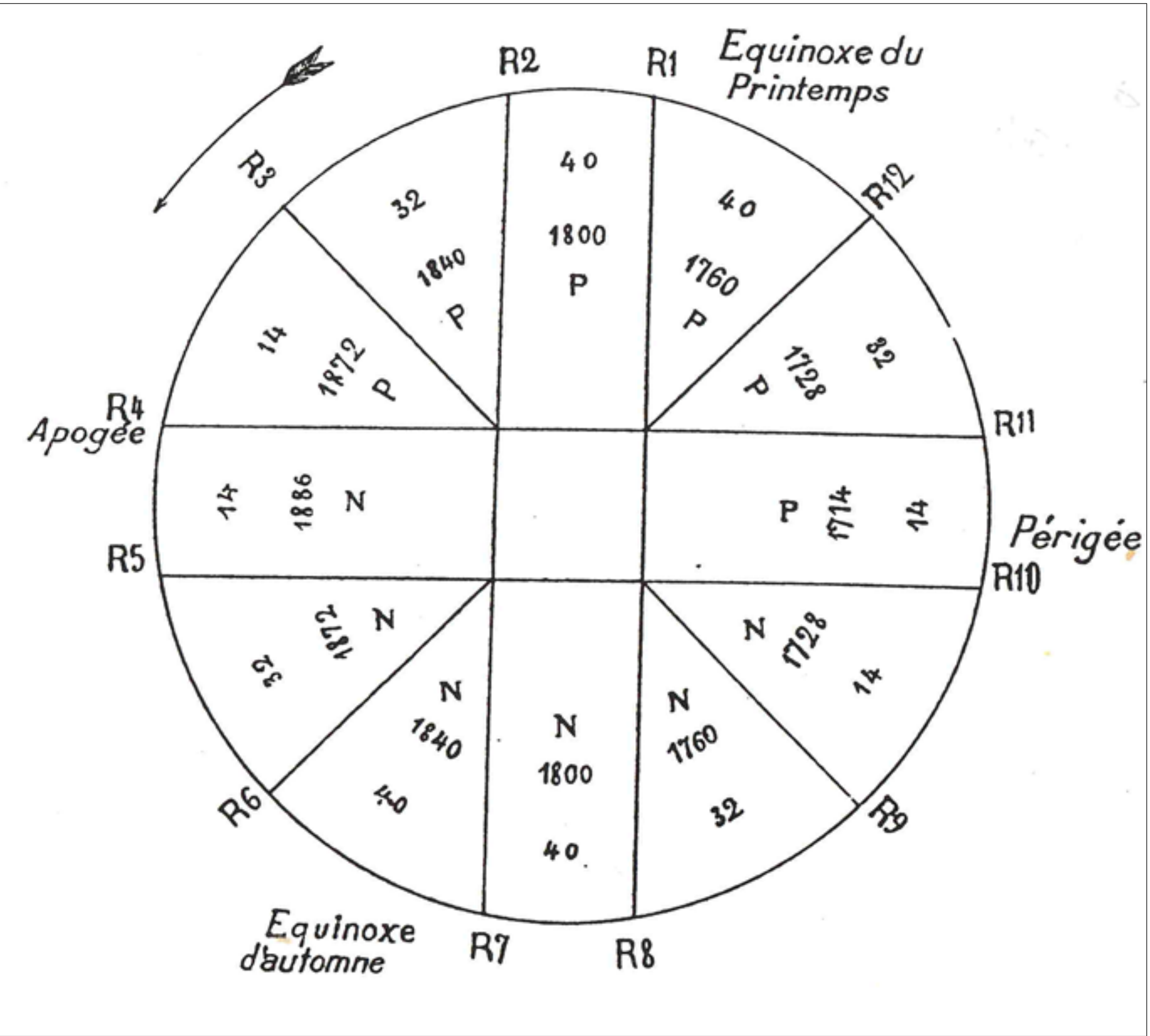

Figure 2: A day length diagram (after Faraut, 1910: 186).

simplifying the computation of the oblique ascension differences and that it has obvious roots in Indian astronomy. It is also very likely that it was used in other parts of Southeast Asia.

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Dr Lars Gislén was born in Lund (Sweden) in 1938, and received a PhD in high energy particle physics from the University of Lund in 1972. He worked in 1970/1971 as a researcher at the Laboratoire de Physique Théorique in Orsay (France) with models of high energy particle scattering. He has also done research on atmospheric optics and with physical modelling of biological systems and evolution.

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