DIAMETERS (*BIMBAS*) OF THE SUN, MOON AND EARTH'S SHADOW-CONE IN INDIAN ASTRONOMICAL TEXTS, WITH SPECIAL REFERENCE TO THE *MAKARANDASĀRIṆĪ* AND THE *GAṆAKĀNANDA*

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Abstract: The diameters of the Sun, Moon and Earth's shadow-cone are important in the computation of lunar and solar eclipses. This paper discusses the procedures for computing the diameters of the Sun, Moon and Earth's shadow-cone according to the *Makarandasāriṇī* and the *Gaṇakānanda* texts. The results are compared with those of the basic *siddhādntic* text, *the Sūryasiddhānta*, and with the popular astronomical handbook, the *Grahalāghava* of Gaṇeśa Daivajña (CE 1520). A number of illustrative examples are provided, along with possible explanations.

Keywords: angular diameters, *bimbas*, *nakṣatrabhoga*, *Grahalāghava*, *Makarandasāriṇī*, *Gaṇakānanda*, *Sūryasiddhānta*, *Makaranda*

1 INTRODUCTION

In astronomy, the sizes of objects in the sky are often given in terms of their angular diameters as seen from the Earth. The angular diameter of an object is the angle the object subtends at the observer as seen from the Earth. These angular diameters play a very important role in the computation of lunar and solar eclipses, conjunctions, occultations and transits.

In Indian classical astronomical texts, the procedures for calculating angular diameters *(bimbas)* are given in different forms in different texts. The majority of the *Siddhāntic* texts give angular diameters in terms of the true daily motions of the Sun and Moon. Some other texts, including astronomical tables like the *Makarandasāriṇī*, determine angular diameters as a function of the duration of the running lunar mansion (*nakṣatrabhoga*), or the anomalies of the Sun and Moon from their respective apogees.

By sustained observations over many centuries Indian astronomers realized that the angle subtended by a heavenly body at the viewer (i.e. the angular diameter) depended on the distance of the body from the viewer, while in turn the angular velocity of the body was also related to its distance. The Siddhāntic astronomers made out, though as a first approximation, that the angular diameter of a body was proportional to its true daily motion in longitude as observed from the Earth. However, for makers and users of almanacs (*pañcāṅga*) the readily available parameters were

(i) the true daily motion in longitude;
(ii) the duration of the lunar mansion (*nakṣatrabhoga* or *nakṣatramāna*);
(iii) anomalies of the Sun and the Moon from their respective apogees (*mandocca*); and
(iv) the duration of the Sun's stay in different zodiacal signs (*rāśi*) i.e. of (sidereal) solar months.

These diameters are expressed in different units in different texts, and the notations and units used in this paper are listed and explained below, in Section 1.1. The famous classical *Siddhāntic* text, the *Sūryasiddhānta,* gives diameters in terms of the linear unit *yojana.* In the *Khaṇḍakhādyaka*, by Brahmagupta, the angular diameters are given in minutes of arc (*kalās*). On the other hand, in the *Karaṇakutūhala* by Bhāskara II and the *Grahalāghava* of Gaṇeśa Daivajña, the unit used for the diameter is the *aṅgula*.

In Sections 2–7 we discuss the procedure for determining angular diameters given in the *Makarandasāriṇī* (MKS), and we compare it with those outlined in the *Sūryasiddhānta* (SS) and the *Grahalāghava* (GL).

1.1 Notations and Units Used in this Paper

This paper is based on Sanskrit astronomical texts and tables and hence the units of angle, distance, time etc. used are also in Sanskrit. The following are some that we frequently use in this paper.

1.1.1 Time

1 day = 60 *ghatis*; 1 *ghati* = 60 *vighatis* = 24 minutes (i.e. 1 hour = 2.5 *ghati*)
1 lunar month = *cāndra māsa* = 30 *tithis* (lunar days)

1.1.2 Angles

Amśa = 1 degree; 1 *amśa* = 60 *kalās* (arc minutes)
1 *kala* = 60 *vikalās* (arc seconds)
Rāśi = zodiacal sign; 1 *rāśi* = 30 degrees; 1 *aṅgula* = 3 *kalās* (i.e. 3 arc minutes) = 60 *pratyaṅgulas*

1.1.3 Angles in the *Sūryasiddhānta*

The *yojana*; 1 *yojana* = 15 *kalās*

1.1.4 Zodiacal Signs and other Sanskrit Astronomical Words

The sidereal equivalents of the zodiacal signs (Aries, Taurus, etc.) are called *Meṣa* or *Vṛṣabha* in Indian terminology. The English equivalents of other Sanskrit words, like *bimba*, *nakṣatrabhoga,* etc., mentioned in this paper are provided when first introduced.

2 IMPORTANCE OF ASTRONOMICAL TABLES

In Indian society, for observances of religious festivals and also for civil purposes, a calendrical almanac called a *pañcāṅga* is a necessity. In the Indian tradition, these astronomical almanacs are published annually according to different *pakṣas* (schools), namely the *Saura, Ārya, Brāhma* and *Gaṇeśa pakṣas.*

These astronomical almanacs are based on traditional treatises (*siddhānta*) like the *Sūryasiddhānta*, the *Brahma siddhānta* etc. Since the direct application of the major texts is cumbersome and tedious for the day-to-day positions of heavenly bodies, the almanacs are compiled annually based on tables (*sāriṇīs*) of different *siddhānta pakṣas*. The popular Indian astronomical tables belong to different schools and are based on different major texts. These different *pakṣas* conformed to the parameters and procedures respectively of the *Sūryasiddhānta*, the *Āryabhaṭīyam* of Āryabhaṭa I (CE 476), the *Brahma sphuṭa siddhānta* of Brahmagupta (CE 628) and the *Grahalāghava* of Gaṇeśa Daivajña (CE 1520). The annual almanacs are computed using different sets of astronomical tables like the *Makarandasāriṇī*, *Gaṇakānanda*, *Pratibhāgī*, *Mahādevī*, *Tithicintāmaṇi*, etc. These Indian astronomical tables are variously referred to as *sāriṇī, padakam, koṣṭhaka* and *vākya*. The major tables of the *Saurapakṣa* are (i) the *Makarandasāriṇī*, (ii) the *Gaṇakānanda*, (iii) the *Pratibhāgī* and (iv*)* the *Tyāgarti* manuscripts.

The *Makarandasāriṇī* (MKS) is the most popular text among the Indian astronomical tables. These tables with explanatory verses (*ślokas*) were composed by Makaranda, the son of Ānanda, at Kāśi in CE 1478. This *sāriṇī* belongs to the *Saurapakṣa*. In fact, the MKS is the most popular among the tables (*sāriṇīs*) of the *Saurapakṣa*, and most contemporary almanac-makers in India adopt the MKS tables when computing *tithi*s (lunar days), *nakṣatra*s (lunar mansions)*, yoga* (combinations of longitudes of the Sun and Moon) and *karaṇas* (half a lunar day), and also for computations of planetary positions and eclipses.

The commentaries on these tables are algorithmic in nature, and these explain only how to use the tables for compiling almanacs.

3 OBTAINING DIAMETERS OF THE SUN, MOON AND EARTH'S SHADOW-CONE ACCORDING TO THE MKS

The following tables are given in the MKS for computing angular diameters of the Sun, Moon and Earth's shadow-cone.

Figure 1 is from the text of the MKS in which the angular diameters of the Sun and the Moon are given. In the case of the Moon the true daily motion in longitude is the argument, and in the case of the Sun the argument is the duration of the solar month. The left part of the top entry in the image provides the Moon's angular diameter, and the right part is that of the Sun. The verse below these portions explains the procedure for their use. These are shown in Tables 1 and 2 and the procedures are explained. In Table 1, the angular diameters of the Moon (*candra bimba*) and the Earth's shadow-cone (*pāta bimba*) are given for the duration of the Moon's stay in a mansion (*nakṣatrabhoga or nakṣatramāna*) over the range from 56 *ghaṭīs to* 66 *ghaṭīs*. Also, the corresponding Moon's daily rate of motion (in arc-minutes) in longitude is given in the last row.

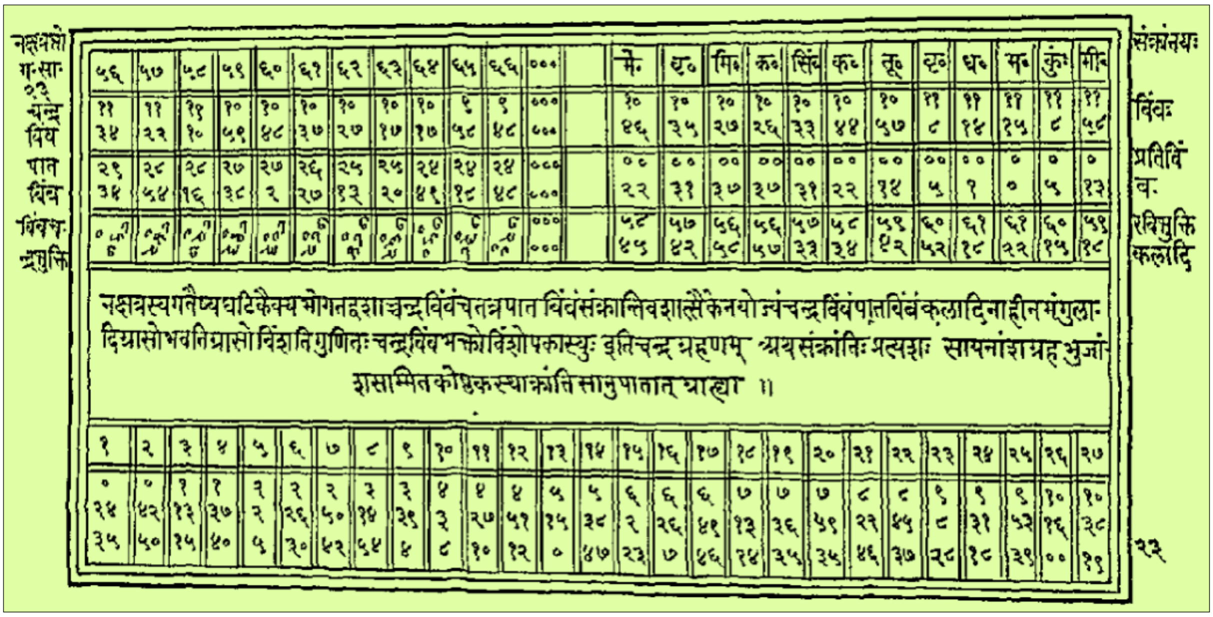

Figure 1: A text from the MKS showing a table of angular diameters for the Sun and Moon.

Table 1: A table for finding the Moon's diameter.

Duration of the lunar mansion (in *ghaṭīs*)	56	57	58	59	60	61	62	63	64	65	66
Moon's diameter (in *aṅgulas*)	11 34	11 22	11 10	10 59	10 48	10 37	10 27	10 17	10 7	9 58	9 48
Diameter of the shadow-cone (in *aṅgulas*)	29 34	28 54	28 16	27 38	27 2	26 27	25 13	25 20	24 49	24 18	24 48
Moon's daily rate of motion (in *kalās*)	857	842	827	813	800	787	774	762	750	738	727

3.1 Rationale for the Moon's Diameter According to the MKS

The angular extent of a lunar mansion (*nakhatra*) = 13° 20′ = 800′
The duration of a lunar mansion (*nakṣatramāna*) = Extent of a *nakṣatra*/Moon's daily motion
= 800/Moon's daily motion (where the Moon's daily motion is in arc minutes per day)
= (800/Moon's daily motion) × 60 *gh*

∴ The Moon's daily motion = 48000/Duration of a *nakṣhatra* [1]

According to the SS,
The Moon's diameter = (Moon's true daily motion/Moon's mean daily motion) × 480, in *yojana*s
= [(48000/Duration of a *nakṣatra*)/790′ 35′′] × 480 using equation [1]
= 29143.03784/Duration of a *nakṣatra*, in *yojana*s
= 647.6230631/Duration of a *nakṣatra*, in *aṅgulas* (converting *yojanas* into *aṅgulas*)
≈ 648/Duration of a *nakṣkatra*, in *aṅgulas*

Note that Table 1 can be generated using the above formula for the Moon's diameter.

Table 2: A table for finding the Sun's diameter.

Solar months	*Meṣa*	*Vṛṣabha*	*Mithuna*	*Karkataka*	*Simha*	*Kanyā*	*Tulā*	*Vṛścika*	*Dhanus*	*Makara*	*Kumbha*	*Mīna*
The Sun's diameter (in *aṅgulas and pratyaṅgulas*)	10 46	10 35	10 27	10 26	10 33	10 44	10 57	11 8	11 14	11 15	11 8	10 58
Corrections for the diameter of the shadow-cone (in *aṅgulas and pratyaṅgulas*)	0 22	0 31	0 37	0 37	0 31	0 22	0 14	0 5	0 1	0 0	0 5	0 13
The Sun's daily motion (in *kalās and vikalās*)	58 45	57 42	56 58	56 57	57 33	58 34	59 42	60 52	61 18	61 22	60 15	59 18

In Table 2 the angular diameter of the Sun (*ravibimbam*) and corrections for the shadow-cone diameter (*pāta bimbas*) are given for 12 solar months. The corresponding daily motion of the Sun (*ravibhukti*) is also given in the last row.

4 THE PROCEDURE FOR FINDING THE DIAMETERS ACCORDING TO THE MKS

The following procedure is given in the MKS for finding the diameters:

(1) Find the duration of the running lunar mansion (*gata-eṣya ghaṱī*) at Full Moon day or New Moon day (*parvānta*) on an eclipse day.
(2) Consider the entry in the column headed by the number represented by the duration of the lunar mansion (taking only the integer part of the *ghaṱī*) corresponding to the row with the Moon's diameter (*candrabimba*).
(3) Find the entry in the next column (next to the column headed the duration of the lunar mansion corresponding to the row with Moon's diameter. This entry is called the *agrimāṅka* (succeeding number).
(4) Find the difference (*agrimāntara*) between the above two entries obtained from steps 2 and 3. Now the remaining fractional part of the duration (in *ghaṱī* or *vighaṱī*) is to be multiplied by this difference and divided by 60. Add or subtract the result obtained to the first value or from *agrimāṅka* (the entries obtained in the steps 2 and 3) respectively.
(5) The above result gives the angular diameter of the Moon (*candrabimba*) in *aṅgulas.*
(6) Similarly, we find the diameter of the Earth's shadow-cone at its intersection with the plane of the lunar orbit (*bhubhābimbam*) using Table 1 and following the same procedure. In this case, instead of the Moon's diameter row, the values corresponding to the row with the diameter of the shadow-cone are to be considered. This diameter of the shadow-cone should be corrected using Table 2, as explained in the following steps.
(7) Find the solar month at *parvānta* by calculating the Sun's position expressed in *rāśi, amśa, kalā* and *vikalās* (sign, degrees, minutes and seconds). Obtain the diameter of the shadow-cone value corresponding to the running solar month and to the next solar month using Table 2, and take the difference between the two values.
(8) Multiply the above difference between the two values by *amśa, kalā* and *vikalās* (degree, minutes and seconds) for the Sun's position considered in the above step 7, and divide the product by 30. This result will be in *aṅgulas*.
(9) Add the above result to the diameter of the shadow-cone value corresponding to the running solar month obtained in step 7. This is the correction factor to be added to the diameter of the shadow-cone obtained using Table 1 in step 6 to get the corrected angular diameter of the Earth's shadow (*bhubhābimbam*).
(10) The same procedure is followed to find the diameter of the Sun using Table 2, corresponding to the solar month at New Moon on the day of the solar eclipse.

4.1 Example Number 1

Given date: *śaka* 1534 *vaiśākha śuddha* 15, Monday, corresponding to CE 15 May 1612.

For the above given date:

Ending moments of the full Moon day (*parvānta ghaṱī*) = 54|40 *gh*
duration of *Anurādhā nakṣatra* = 58|36 *gh*
Anurādhā is the name of the 17th lunar mansion in the list of the 27 lunar mansions of the zodiac.
The true position of the Sun = 1^R 6º 30′ 37″ The true position of the Moon = 7^R 6º 34′ 35″
Rāhu (the Moon's ascending node) = 1^R 14º 18′ 11″
Note: In the above, (i) 54|40 *gh* means 54 *ghatis*, 40 *vighatis*, where 1 *ghati* = 60 *vighatis* = 24 minutes. 58|36 *gh* is also similarly understood. In 1^R 6º 30′ 37″, the notation 1^R = 1 zodiac sign = 30 degrees.

4.1.1 To Find the Diameter of the Moon

Now from Table 1 the entry corresponding to the Moon's diameter row in the column headed by duration number 58 is 11|10
The entry corresponding to the Moon's diameter row in the column headed by next number 59 (succeeding number) is 10|59
The difference = 10|59 – 11|10 = –0|11
The remaining fractional part of the duration = 36
Now, (difference × remainder)/60 = [(–11) × 36]/60 = –396/60 = –6.6 = –6|36
= –0|6|36 *aṅgulas*

The diameter of the Moon (*candrabimba*) = 11|10 + (–0|6|36) = 11|4 *aṅgulas*

4.1.2 To Find the Diameter of the Earth's Shadow-cone

From Table 1 the entry corresponding to the diameter of the shadow-cone row in the column headed by duration number 58 is 28|16
The entry corresponding to the diameter of the shadow-cone row in the column headed by next number 59 (succeeding number) is 27|38
The difference = 27|38–28|16 = –0|38
(difference × remainder)/60 = [(–38) × 36]/60 = –22|48 ≈ 22
The diameter of the shadow-cone (*bhubhābimbam*) = 28|16 + (–0|22) = 27|54 *aṅgulas*

4.1.3 Correction to the Diameter of the Shadow-cone using Table 2

The true position of the Sun at Full Moon (*parvānta*) = 1^R 6° 30′ 37″
The number 1 in the *rāśi* position indicates that the current solar month is *Vṛṣabha*, and the remaining degree etc. in the Sun's position is 6° 30′ 37″
Now, the entries in the column of *Vṛṣabha* and *Mithuna* corresponding to the diameter of the shadow-cone are 0|31 and 0|37 respectively.
The difference = 0|37–0|31 = 0|6
(difference × remainder)/30 = (6 × 6° 30′ 37″)/30 = 1.3020556 ≈ 1 = 0|1 *aṅgulas* (ignoring the fraction)
The correction to the diameter of the shadow-cone = 0|31 + 0|1= 0|32 *aṅgulas*
The corrected diameter of the shadow-cone = 27|54 + 0|32 = 28|26 *aṅgulas*

4.2 Example 2: To Find the Diameter of the Sun

Given date: *śaka* 1532 *Mārgaśira kṛṣṇa* 30 Wednesday corresponding to CE 15 December 1610
Ending moments of the new Moon day (*parvānta ghaṭī*) = 11|59
The true position of the Sun = 8^R 5° 26′ 20″
The number 8 in the *rāśi* position indicates that the current solar month is *Dhanu* and the next is *Makara.*
The entries in the column headed by *Dhanu* and *Makara* corresponding to the diameter of the Sun in Table 2 are 11|14 and 11|15 respectively.
Their difference = 11|15–11|14 = 0|1
Now the Sun's diameter = 11|14 + (difference × remainder)/30
= 11|14 + (0|1 × 5° 26′ 20′′)/30 = 11|14 + 0|10|52
= 11|24|52 ≈ 11|25 *aṅgulas*
Remark: The dates given in the above examples are taken from Viśvanātha's commentary on the MKS. The same examples of eclipses are given by Viśvanātha in his commentary on the very popular text *Grahalāghava* of Gaṇeśa Daivajña.

4.3 Example 3: A Modern Example

Given date: Tuesday 16 July 2019
There is a partial lunar eclipse visible in India on the given date.
Ending moments of Full Moon day (*parvānta*) = 52|22 *gh*
Duration (*gata eṣya ghaṭī*) of *Pūrvāṣāḍha nakṣatra* (20^{th} lunar mansion = 64|37.5 *gh*)
True position of the Sun = 3^R 24° 3′ 56″
True position of the Moon = 9^R 24° 3′ 55″
The Moon's node (*Rāhu*) = 3^R 17° 7′ 48″

4.3.1 To Find the Diameter of the Moon

Now from Table 1 the entry corresponding to the diameter of the Moon row in the column headed by duration (in *ghaṭī*) number 64 is 10|7
The entry corresponding to the diameter of the Moon row in the column headed by the next number 65 (succeeding number) is 9|59
The difference = 9|59–10|7 = –0|8
The remaining fractional part of the duration (in *ghaṭī* or *vighaṭī*) = 37.5
Now, (difference × remainder)/60 = [(–8) × 37.5]/60 = –300/60 = –0| 5|37.5
≈ –0|6 *aṅgulas*
The diameter of the Moon (*candrabimba*) = 10|8 + (–0|6) = 10|2 *aṅgulas*

4.3.2 To Find the Diameter of the Earth's Shadow-cone

From Table 1 the entry corresponding to the shadow diameter row in the column headed by duration

(in *ghaṭī*) number 64 is 24|49
The entry corresponding to the shadow diameter row in the column headed by the next number 65 (the succeeding number) is 24|18
The difference = 24|18–24|49 = –0|31
(difference × remainder)/60 = [(–31) × 37.5]/60 = –19|22 ≈ 19
Diameter of the shadow-cone (*bhubhābimbam*) = 24|49 + (–0|19) = 24|30 *aṅgulas*

4.3.3 Correction to the Diameter of the Shadow-cone Using Table 2

True position of the Sun at full Moon = 3^R 24° 3′ 56″
The number 3 in the *rāśi* position indicates that the current solar month is *Mithuna* and the remaining degree etc, in the Sun's position is 24° 3′ 56″
Now, the entries in the column of *Mithuna* and *Karkataka* solar months corresponding to the shadow diameter are 0|37 and 0|37respectively.
The difference = 0|37– 0|37= 0|0
(difference × remainder)/30 = 0|0 (ignoring the fraction)
Correction to the diameter of the shadow-cone = 0|0 *aṅgulas*
Corrected diameter of the shadow-cone = 24|30+0|0 = 24|30 *aṅgulas*

4.4 Example 4: To Find the Diameter of the Sun

Given date (modern example): 26 December 2019
This is an annular solar eclipse visible in India on the given date.
Ending moments of New Moon day (*parvānta ghaṭī*) = 11|59
True Sun = 8^R 9° 58′ 57″
The number 8 in the *rāśi* position indicates that the current solar month is *Dhanu* and the next is *Makara.* The entries in the column headed by *Dhanu* and *Makara*.
The corresponding diameters of the Sun in Table 2 are 11|14 and 11|15 respectively.
Their difference = 11|15–11|14 = 0|1
Now the Sun's diameter = 11|14 + (difference × remainder)/30
= 11|14 + (0|1 × 3^R 24° 3′ 56″)/30 = 11|14 + 0|0|19.97
= 11|14|19.97 ≈ 11|14 *aṅgulas*

5 THE ANGULAR DIAMETER OF THE SUN ACCORDING TO KUPPANNA SASTRY

Professor T.S. Kuppanna Sastry (1900–1982) was a highly respected Sanskritist and erudite in both traditional and modern astronomical theories and procedures. Professor Sastry devoted all his time to a critical study and appreciation of almost all *sāstras* including *Jyotiṣa* (Astronomy), *gaṇita* (Mathematics) and modern astronomy. He has edited six astronomical texts. He brought out a critical edition of the *Mahābhāskarīya* with the commentaries of Govindsvāmin and Parameśvara with annotations and indices in 1957. He edited the *Vākyakaraṇa* with the commentary of Sundararāja in 1962. He also critically edited the *Vedāṅgajyotiṣa* and *Pañcasiddāntikā* with translations and notes. Professor Sastry's immensely scholarly and useful "Collected papers on *Jyotiṣa*" was brought out by Rashtriya Sanskrit Vidya Peetha (now the National Sanskrit University), in Tirupati.

According to Kuppanna Sastry, the Sun's angular diameter is given by 2[961.2 + (16.1)cos(GS)] in *kalās* (seconds of arc), where GS is the Sun's anomaly from its perigee. Table 3 gives the Sun's diameter at different *sankrānti* (the Sun's entry into 12 *rāśis*) calculated using Kuppanna Sastry's formula in both *kalās* (seconds of arc) and *aṅgulas*.

Table 3: The Sun's diameter using Kuppanna Sastry's formula.

Sl. No.	Solar month	Mean Sun at beginning of month (°)	GS (°)	Sun's diameter in *kalās*	Sun's diameter in *aṅgulas* and sub units
1	*Meśa*	0	–102	31.92842073	10l38l34
2	*Vṛśabha*	30	–132	31.68089991	10l33l37
3	*Mithunna*	60	–162	31.52959967	10l30l35
4	*Karkataka*	90	–192	31.51506079	10l30l18
5	*Simha*	120	–222	31.64117894	10l32l49
6	*Kanyā*	150	–252	31.87416088	10l37l29
7	*Tulā*	180	–282	33.15157927	10l43l02
8	*Vṛścika*	210	–312	32.39910009	10l47l59
9	*Dhanu*	240	–342	32.55040033	10l51l0.48
10	*Makara*	270	–372	32.56493921	10l51l18
11	*Kumbha*	300	–402	32.43882106	10l48l47
12	*Mīna*	330	–432	32.20583912	10l44l07

According to the above table, the Sun's diameter is minimum at *Karkataka*, that is 10|30|18 *aṅgulas*, and is maximum at *Makara*, i.e. 10|51|18 *aṅgulas*.

6 DIAMETERS OF THE SUN, MOON AND EARTH'S SHADOW ACCORDING TO THE SS AND GL

In the *Sūryasiddhānta* (SS) the diameters are expressed in linear units called *yojanas.* In this text, the mean diameters of the Sun, the Moon and the Earth's shadow-cone are taken as 6500, 480 and 1600 *yojanas* respectively. The true diameters are as given below.

Sun's diameter = (Sun's true daily motion × 6500/Sun's mean daily motion) × (43,20,000/577,53,336) *yojanas*
Moon's diameter = (Moon's true daily motion × 480/Moon's mean daily motion) *yojanas*
Shadow-cone diameter = (Moon's true daily motion × 1600/Moon's mean daily motion) – [(Sun's diameter – 1600) × 480/6500] *yojanas*

Note that in the SS, the revolutions of the Sun and the Moon in a *Mahāyuga* of 432 × 10^4 years are respectively taken as 43,20,000 and 577,53,336. Note, also, that 1 *yojana* = 15 *kalās,* 1 *aṅgula* = 3 *kalās*, 1 *kalā* = 60 *vikalās* and 1 *aṅgula* = 60 *pratyaṅgulas*.

In Table 4 we give the diameter of the Sun according to the MKS, SS, and Kuppanna Sastry's formulae, for different solar months. While calculating using the SS formula we have used the Sun's daily motion for different solar months given in the MKS, as the Sun's true daily motion and mean daily motion of the Sun are taken as 58′ 58″ per mean sidereal day.

Table 4: The Sun's diameter according to the MKS, SS and Kuppanna Sastry.

Sl. No.	Solar month	Sun's motion according to the MKS (in *kalās*)	Sun's diameter according to the MKS (in *aṅgulas*)	Sun's diameter according to the SS (in *aṅgulas*)	Sun's diameter according to Kuppanna Sastry (in *aṅgulas*)
1	*Meśa*	58|45	10|46	10|45|53	10|38|34
2	*Vṛṣabha*	57|42	10|35	10|34|21	10|33|37
3	*Mithunna*	56|58	10|27	10|26|17	10|30|35
4	*Karkataka*	56|57	10|26	10|26|06	10|30|18
5	*Simha*	57|33	10|33	10|32|42	10|32|49
6	*Kanyā*	58|34	10|44	10|43|52	10|37|29
7	*Tulā*	59|42	10|57	10|56|20	10|43|02
8	*Vṛścika*	60|22	11|08	11|03|40	10|47|59
9	*Dhanu*	61|18	11|14	11|13|55	10|51|0.48
10	*Makara*	61|22	11|15	11|14|40	10|51|18
11	*Kumbha*	60|15	11|08	11|02|23	10|48|47
12	*Mīna*	59|18	10|58	10|51|56	10|44|07

From Table 4 we observe that the values according to the MKS and SS coincide up to a *pratyaṅgula*. But those based on Kuppanna Sastry's expression are less by 10 to 12 *pratyaṅgulas.*

In the *Grahalāghava* (GL) the angular diameters are expressed in *aṅgulas.* The expressions given in the GL for finding diameters are as follows:

Sun's diameter = [(Sun's true daily motion – 55)/5] + 10, in *aṅgulas*
Moon's diameter = Moon's true daily motion/74, in *aṅgulas*
Shadow diameter = [(3/11) × Moon's diameter + 3 × Moon's diameter] – 8, in *aṅgulas*

In Table 5 below we have compared the diameter of the Moon according to the three texts, namely the MKS, SS and GL. The Moon's angular diameter is computed by considering the duration of a lunar mansion (*nakṣatrabhoga*) and the Moon's daily motion corresponding to the duration of a lunar mansion (*bimba candra bhukti*) given by the MKS in SS and GL formulae.

Table 5: The Moon's diameter according to the MKS, SS and GL.

Sl. No.	Duration of lunar mansion (in *ghaṭīs*)	Moon's daily motion (in *kalās*)	Moon's diameter according to the MKS (in *aṅgulas*)	Moon's diameter according to the SS (in *aṅgulas*)	Moon's diameter according to the GL (in *aṅgulas*)
1	56	857	11|34	11|35|40	11|34|52
2	57	842	11|22	11|23|29	11|22|42
3	58	827	11|10	11|11|19	11|10|32
4	59	813	10|59	10|59|57	10|59|11
5	60	800	10|48	10|49|24	10|48|39
6	61	787	10|37	10|38|51	10|38|00
7	62	774	10|27	10|28|17	10|27|34
8	63	762	10|17	10|18|33	10|17|50
9	64	750	10|7	10|08|49	10|08|06
10	65	738	09|58	09|59|04	09|58|22
11	66	727	09|48	09|50|08	09|49|27

Note: In the MKS the diameters are given only in *aṅgulas* and *pratyaṅgulas*, with no further subunits.

From Table 5 we notice that the values for the Moon's diameter according to the MKS and the GL coincide within one *pratyaṅgula* although they belong to different schools.

7 DIAMETERS ACCORDING TO THE *GAṆAKĀNANDA*

The *Gaṇakānanda* is a popular text in the Andhra and Karnataka regions. The epocal date of the text is CE 16 March 1447, and it is based on the *Sūryasiddhānta*. The Telugu translation by Chella Lakshmi Nrusimha Sastrigaru of Machlipatnam is taken up. It is a handbook (*karaṇa* text) comprising a textual part and astronomical tables. The famous Andhra astronomer Sūrya, son of Bālāditya, composed his famous *karaṇa* (tables) called the *Gaṇakānanda.* His more illustrious protégé, Yallaya, composed his exhaustive commentary the *Kalpavallī*, which was based on the well-known *Sūrya-siddhānta.*

Yallaya belonged to the *kāśyapa gotra* (lineage) and his genealogy was as follows: Kalpa Yajvā (great grandfather) → Yallaya (grandfather) → Śrīdhara (father) → Yallaya. Yallaya learnt three works from his teachers: the *Gaṇakānanda*, the *Daivajñābharaṇa* and the *Daivajñabhūṣaṇa*. Yallaya's residence was a small town to the north of Addankī (latitude 15°.49 N, longitude 80°.01 E) called Skandasomeśvara in Andhra Pradesh. Yallaya's home town lay towards the āgneya (south-east) of Śriśaila, the famous pilgrimage centre.

The following are the abbreviations for the symbols used: *SDIA* = the Sun's angular diameter; *STDM* = the Sun's true daily motion; and *SMDM* = the Sun's mean daily motion.

The true daily motions of the Sun and the Moon, *STDM* and *MTDM* vary with their respective *manda* anomalies (*Mandakendra*, *MK*). The variation of the Sun's true daily motion with its *MK* is given by:

$STDM = 59.1388 - (1.98109) \cos (MK) - (0.041285) \cos (2MK)$ in *kalās* (arc min.) [2]

Here, the *manda* anomaly MK = Sun's *Mandocca* (apogee) – Mean Sun. Similarly, the Moon's true daily motion, in terms of its *manda* anomaly *MK*, is given by

$MTDM = 790.5666 - (PRD/360) \times 783.9 \times \cos (MK)$ in *kalās* (arc min.) [3]

where *PRD* is the Moon's *manda paridhi* (periphery). Here also, *MK* = Moon's *Mandocca* (Apogee) – Mean Moon. From [2] and [3], as *MK* varies from 0° to 360°, the minimum values of the true daily motions (*TDM*) of the Sun and the Moon are as shown in Table 6.

Table 6: Minimum and maximum true daily motions.

Body	Minimum *TDM*	Maximum *TDM*
Sun	56′.83	61′.43
Moon	720′.8866	860′.2466

The variations of the Sun's true daily motion (*STDM*) and its diameter (*Ravi bimba*) *SDIA* with the *MK* are shown in Table 7. Similarly, the variations of the Moon's true daily motion (*MTDM*) and its diameter *MDIA*, *Candra bimba*) are shown in Table 8.

Table 7: The Sun's angular diameter according to the *Sūryasiddhānta*.

MK	STDM (')	SDIA (')
0	56.83367	31.24122
10	56.88734	31.27071
20	57.00755	31.33679
30	57.18918	31.43664
40	57.4256	31.56659
50	57.70905	31.7224
60	58.0309	31.89933
70	58.38198	32.09231
80	58.7527	32.2961
90	59.1333	32.50531
100	59.5139	32.71452
110	59.88463	32.91831
120	60.23571	33.1113
130	60.55756	33.28822
140	60.84101	33.44403
150	61.07743	33.57399
160	61.25906	33.67383
170	61.37927	33.73991
180	61.43293	33.76941

Table 8: The Moon's angular diameter according to *the Sūryasiddhānta*.

MK	MTDM (')	MDIA(MOD)'	MDIA(SS)'
0	720.8866	29.28333	28.88662
10	722.1935	29.28333	28.88662
20	725.5553	29.31428	28.93899
30	730.8505	29.40616	29.0737
40	737.9034	29.55619	29.28588
50	746.4919	29.75982	29.5685
60	756.3551	30.01086	29.91265
70	767.2012	30.30167	30.30787
80	778.715	30.62342	30.74249
90	790.5666	30.96634	31.20386
100	802.4181	31.32	31.67876
110	813.9321	31.67367	32.15367
120	824.7781	32.01658	32.61504
130	834.6413	32.33833	33.04965
140	843.2299	32.62914	33.44488
150	850.2827	32.88018	33.78903
160	855.5779	33.08381	34.07165
170	858.9398	33.23384	34.28383
180	860.2466	33.32573	34.41854

Table 9: The angular diameter of the Earth's shadow-cone according to the *Sūryasiddhānta*.

STDM (') → MTDM (')↓	56	57	58	59	60	61	62
720	74.9	74.4	73.8	73.3	72.7	72.2	71.6
730	76.3	75.7	75.2	74.6	74.1	73.5	73
740	77.6	77.1	76.5	76	75.4	74.9	74.3
750	79	78.4	77.9	77.3	76.8	76.2	75.7
760	80.3	79.8	79.2	78.7	78.1	77.6	77.1
770	81.7	81.1	80.6	80	79.5	78.9	78.4
780	83	82.5	81.9	81.4	80.8	80.3	79.8
790	84.4	83.8	83.3	82.7	82.2	81.7	81.1
800	85.7	85.2	84.6	84.1	83.5	83	82.5
810	87.1	86.5	86	85.4	84.9	84.4	83.8
820	88.4	87.9	87.3	86.8	86.3	85.7	85.2
830	89.8	89.2	88.7	88.2	87.6	87.1	86.5
840	91.1	90.6	90	89.5	89	88.4	87.9
850	92.5	91.9	91.4	90.9	90.3	89.8	89.2
860	93.8	93.3	92.8	92.2	91.7	91.1	90.6
870	95.2	94.6	94.1	93.6	93	92.5	91.9

The angular diameter of the Earth's shadow-cone (*SHDIA*) varies with the true daily motions of both the Sun and the Moon (*STDM* and *MTDM*), as shown in Table 9, while the *STDM* varies from a minimum of 56′.83 to a maximum of 61′.43. The corresponding external values of the *MTDM* in the case of the Moon are respectively 720′.8866 and 860′.2466 (see Table 8).

Table 10: Diameters of the Moon, the Earth's shadow-come and the Sun according to the MKS, SS, GL and modern calculations for different dates of lunar and solar eclipses.

Lunar Eclipse													
No	Date	Moon's diameter (in *aṅgulas*)				Shadow-cone diameter (in *aṅgulas*)				Sun's diameter (in *aṅgulas*)			
		MKS	SS	GL	Modern	MKS	SS	GL	Modern	MKS	SS	GL	Modern
1	15 May 1612	11|4	10|56	10|55	10|33.7	28|26	28|38	27|44	28|17	10|33	10|32	10|30	10|32.98
2	16 July 2019	10|2	9|57	9|58	9|59|40	24|34	25|33	24|37	26|34	10|20	10|29	10|26	10|30
Solar Eclipse													
		Sun's diameter (in *aṅgulas*)				Moon's diameter (in *aṅgulas*)							
		MKS	SS	GL	Modern	MKS	SS	GL	Modern				
3	15 Dec 1610	11|24.8	11|14	11|15	10|51.4	9|44.5	9|45	9|49	9|48				
4	26 Dec 2019	11|25	11|12	11|14	10|51	10|44	10|38	10|37	10|01				

8 CONCLUSION

In the present paper we have discussed in detail the procedures for determining the angular diameters of the Sun, the Moon and the Earth's shadow according to the *Makarandasāriṇī* and the *Gaṇakānanda*, and presented concrete examples. Also, the results using the MKS were compared with those according to the SS, GL and Kuppanna Sastry (e.g. see Table10).

The specialty of the MKS lies in giving the procedure for obtaining the angular diameters of the Sun, Moon and Earth's shadow-cone using the total duration of the running lunar mansion and the solar months, values of which are readily available in traditional astronomical almanacs (*pañcāṅga*).

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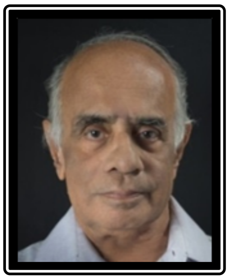

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