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SEVENTEENTH CENTURY FRENCH JESUIT LONGITUDE DETERMINATIONS IN ASIA: ON THE ART OF RECTIFYING THE CLOCKS

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Abstract: Two methods of setting clocks to show correct time during the 17th century by French Jesuits in China are investigated and their computations and accuracy are analysed. These methods were used to rectify the clocks used for timings of eclipses of the Galilean moons or for lunar eclipses that in turn were used to determine geographical longitudes by Jesuit missionaries in Siam and China. It is found that the calculations needed, in spite of being quite complicated, were well done and very accurate.

Keywords: time, clock, longitude, Jesuits, China.

1 INTRODUCTION

In the seventeenth century logitude determinations were of great scientific and commercial interest and demand as the future colonial states of Europe started to explore new routes to countries in the Far East and the Americas. Longitude can be measured by timing an event that is simultaneous for different locations on the Earth but will occur at different local times. The time difference is proportional to the difference in longitude, such that 1 hour of time difference corresponds to 15° of longitude difference.

There are different kinds of events that were suitable at the time, one of which was timing ingresses and egresses of the then discovered satellites of Jupiter behind or in the shadow of the mother planet. These events were carefully observed and timed by observatories in England and France and the renowned astronomer Giovanni Cassini (1625–1712) in France made tables and ephemerides that made it possible to calculate and predict the times of these events. Lunar eclipses were more rare events that could be used. Also the method of measuring lunar distances (de Grijs, 2020) from reference stars could be used but there is no indication the this method was ever used by the French Jesuits in China. The French Jesuit Guy Tachard (1651–1712), determined the longitude of Cape Town timing the eclipse of the Jovian moon Io in the night of 2 June 1685 and of Lop Buri in Siam (present-day Thailand) using the lunar eclipse of 11 December 1685 (see Gislén, 2004; Gislén et al., 2018; Orchiston et al., 2016; Tachard, 1981). An even rarer event of this kind is documented in the records, the transit of Mercury on 10 October 1689, which was observed and timed from Canton, China, by Father Jean de Fonteney (1643–1710) at about 3 o'clock in the afternoon (Anonymous, 1729: 825).

During the seventeenth and eighteenth centuries, the time used in astronomy was what we today call apparent local solar time: a correct clock showing apparent time at the specific location would show 12 hours at noon when the Sun was precisely due south. The time used today is mean solar time based on a fictive mean Sun. The difference between the apparent and mean time is called the equation of time, but it will not appear in this paper as mean time is not used. The reason for using apparent local time was that the mechanical clocks of the time were quite unreliable, sometimes being fast or slow by several minutes or more per day.

There was then a need to rectify the clocks at regular intervals to ensure that they were in phase with the Sun. The most direct way was then to use the true Sun as the timekeeper. In order to have the required precision in the longitude determinations it was necessary to have correct timings of astronomical events with an accuracy of a fraction of a minute. There were two different methods in use by the French Jesuits in China at the time for rectifying a clock as described in detail below.

French Jesuit astronomer Father Fonteney was asked by King Louis XIV to set up a mission to China in order to spread French and Catholic influence at the Chinese court. Fonteney assembled a group of five other Jesuits to accompany him, all highly skilled in the sciences: Joachim Bouvet (1656–1730), Jean-François Gerbillon (1654–1707), Louis-Daniel Lecomte (1655–1728), Claude de Visdelou (1656–1737) and the afore-mentioned Guy Tachard. Before setting out for their destination in 1685, they were admitted to the

Royal French Academy of Sciences and were trained and commissioned to carry on astronomical observations in order to determine the geographical positions of the various places they were to visit and to collect various scientific data, one of the important tasks of the Jesuit team.

The Jesuit Fathers, after being provided with all necessary scientific instruments and up-to-date tables[1] from Paris Observatory, sailed from Brest, in the morning of 3 March 1685, with Père Fonteney as leader (de Chaumont, 1733; Tachard, 1686). They were on board the King's warship *Oiseau* and escorted by the royal frigate *Maligne*. On board were also the Ambassador of the French King to Siam, Chevalier Alexandre de Chaumont (1640–1710) and Abbé François-Timoléon de Choisy (1644–1724) both of whom later wrote accounts of their voyage (de Chaumont, 1733; de Choicy, 1741). The ships made a stop for a week in Cape Town and then continued their voyage. After spending some time in Siam, where Tachard remained, the group continued and finally arrived in Beijing on 7 February 1688. The Jesuits were well received by the Kangxi Emperor, 康熙 (1662–1723; Figure 1), third Emperor of the Qing Dynasty, who had been favourably impressed by Western science and earlier visits by Europeans. Fathers Bouvet and Gerbillon stayed in Beijing, teaching the Emperor mathematics and astronomy, while the other Jesuit astronomers moved to different locations in China.

2 THE SETTING METHODS

The first method was timing and measuring the altitude, *a*, of a reference star. The altitude was corrected for refraction. Needed was also the geographical latitude, ϕ, of the site, measured beforehand and the declination of the star, δ, found in a table. The geographical latitude was determined by measuring of the altitude of the upper limb of the Sun when it passed the meridian and then correct the altitude for refraction and for the semi-diameter of the Sun in order to determine the true altitude of the centre of the Sun, *A*. With the known current declination, *D*, of the the Sun, the geographical latitude was then calculated by the simple formula $\phi = 90° - (A - D)$. Also altitude measurements of stars could in the same way be used to determine the latitude. Fonteney, for example, made 17 such altitude determinations with a 26-inch quadrant between 25 April and 28 December 1689 from the north-west Chinese town Si-nghan-fu (modern Xi'an in the province of Shensi/Shaanxi) in order to determine the latitude of the site. His average value 34° 16′ 30″ N, is very close to the modern value of 34° 17′.

The hour angle, *h*, of the star is the angle between the meridian plane and a plane defined by the local vertical and the direction to the star. The hour angle is zero when the star is in the meridian and counted positive if the star is west of the meridian. We have a relation between these quantities

$$\sin a = \sin\phi \sin\delta + \cos\phi \cos\delta \cos h \qquad (1)$$

from which we can solve for the hour angle *h* once the other quantities are known.

Figure 1: The Kangxi Emperor in Court dress (https://en.wikipedia.org/wiki/Yongzheng_Emperor#/media/File:Portrait_of_the_Yongzheng_Emperor_in_Court_Dress.jpg).

With α, the right ascension of the star taken from a table, the sidereal time Θ is given by

$$\Theta = \pm h + \alpha \qquad (2)$$

The solar right ascension, *as*, that is not constant but slowly varying, can be interpolated from a table for the time of the day and the sidereal time can be eliminated and we finally get the solar hour angle *hs* in known quantities,

$$hs = \Theta - \alpha s = \pm h + \alpha - as \qquad (3)$$

The method can also be used for the Sun itself when $\alpha = \alpha s$ and simply $hs = h$.

As the solar right ascension was given to the Jesuits in the form of day tables for the longitude of Paris, they had to interpolate for the local time of their observation site that corresponded to the local Paris time. In order to do that they needed to already have some idea of what the longitude difference was or without that had to make successive approximations, for instance by first using the right ascension for their local time, calculate a longitude difference, correct the right ascension for the time difference and so on until their result converged. The Jesuits in China seem to have applied a standard time difference of 7 hours for this.

Comparing the calculated solar hour angle (converted to time from noon by multiplying by 6 and dividing by 90) with the time of the clock will then give the clock correction. Using this method with a star is obviously restricted to night time.

The second setting method uses timings of two equal altitudes of the Sun, one before, the other after noon and corrected for refraction. Normally, the altitude of the upper limb of the Sun was used but for example Tachard in Cape Town used the upper solar limb in the morning and the lower limb in the afternoon. He then had to correct for the diameter of the solar disk. The telescope used for observation could swing around a vertical axis between the two azimuthal directions and had to be carefully set up and levelled. Noon would at first be assumed to be in the middle between the two timings. However, the declination of the Sun changes a little between the two timings and the time after noon has to be corrected for that.

Father Fonteney describes his method (translated from Anonymous, 1729: 860):

> Of all the methods that one uses to correct the clock by observations of the Sun, observed before and after noon, I have chosen the following as I am more used to it than to other methods.
>
> I take the difference between the times of observation in the morning and in the afternoon. I change the half of this difference to degrees of the parts of the great circle that gives me how much the Sun, at the morning observation, is distant from the meridian, more or less precisely. With this distance [*h*], the complement of the altitude of the pole (90°– ϕ) and of the corrected altitude (90°– *a*) of the upper limb of the Sun, I find what is called the solar angle (ξ), by this analogy:
>
> As the sine of the complement of the corrected altitude of the Sun is to the sine of the complement of the altitude of the pole; so is the sine of the distance of the Sun from the meridian (the hour angle) to the solar angle. (Rule 1)
>
> I then take the difference of the declination of the Sun in 24 hours on the day of observation of which I take the part of the difference in declination proportional to the interval of observation before and after noon, to which, as the Sun describes a parallel with the equator, I add (i.e. divide by cos δ) the proportion coming from difference between the equator and the parallel of the day: and with this difference of declination increased in this way I have:
>
> As the sine of the solar angle is to the part of the difference in declination, proportional to the interval between the observations, increased by the proportion of the equation to the parallel of the day: so is the sine of the complement of the solar angle to parts of 360° (hour angle, which, reduced to time measure, gives the correction to the time of observation in the afternoon. (Rule 2)

3 MATHEMATICAL FORMULATION

Fonteney's rules can be formulated in mathematical language using spherical geometry and Figure 2 to derive his rules.

Z is the local zenith, *P* the Northern Celestial Pole. *S* and *S'* are the locations of the Sun with slightly different declinations but the same altitude. *ZS* = *ZS'* are zenith distances of the Sun. The great circle through *PZ* is the meridian. *SS'* is part of a local equal altitude parallel circle. *S'A* is a part of a declination parallel circle. Halving the time interval between the morning and afternoon timings will give a first approximation of the hour angle *h*. The angle *ZPS* is the afternoon hour *h* angle of the Sun. The angle ZSP is the solar angle ξ. Δh is the correction to the hour angle due to change in the declination of the Sun between the timings. *SA* is the change in declination $\Delta\delta$ between the two timings. *S'A* is the projection of the hour angle change on the declination parallel: $\Delta h \cos \delta$.

For small changes we can consider the triangle *SS'A* to be a plane triangle. The angle *SS'A* is equal to ξ.

The sine theorem applied to the spherical triangle *ZSP* gives Rule 1 above:

$$\sin h / \cos a = \sin \xi / \cos \phi \qquad (4)$$

With the hour angle *h*, the altitude *a*, and the geographical latitude ϕ known, this deter-

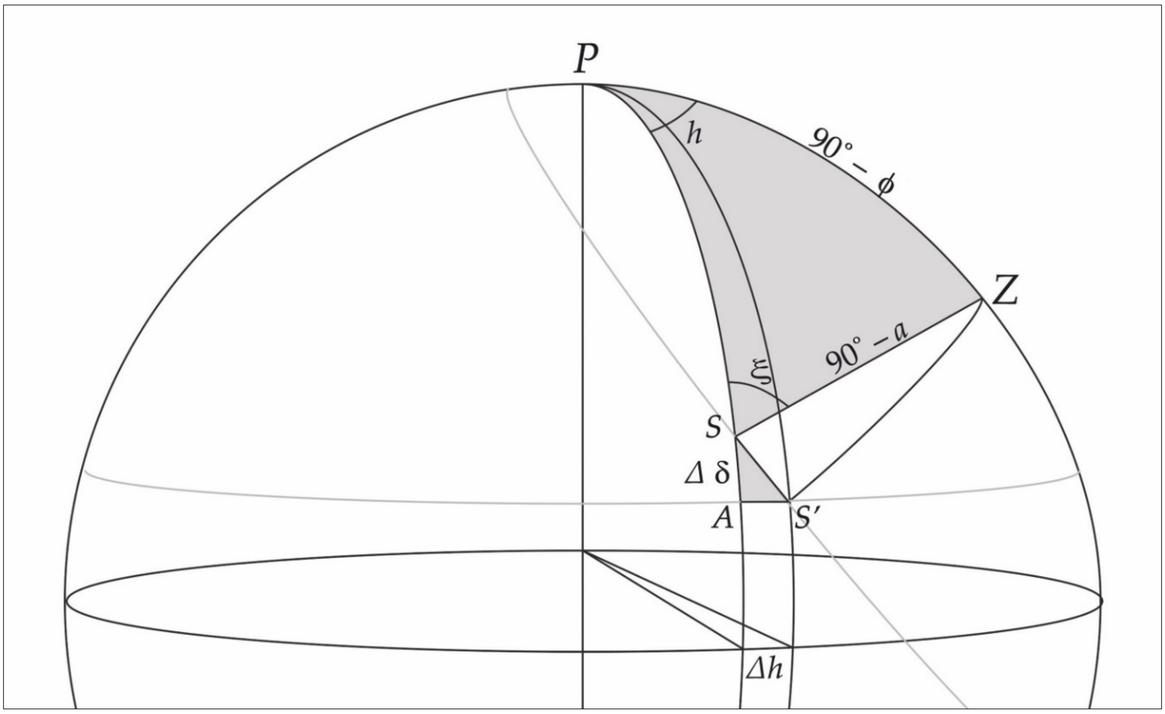


Figure 2: The celestial sphere (diagram: Lars Gislén).

mines the solar angle ξ.

Triangle *SS′A* gives Rule 2:

$\sin \xi / \Delta\delta = \cos \xi / (\Delta h \cos \delta)$

We can now solve for the correction to the hour angle:

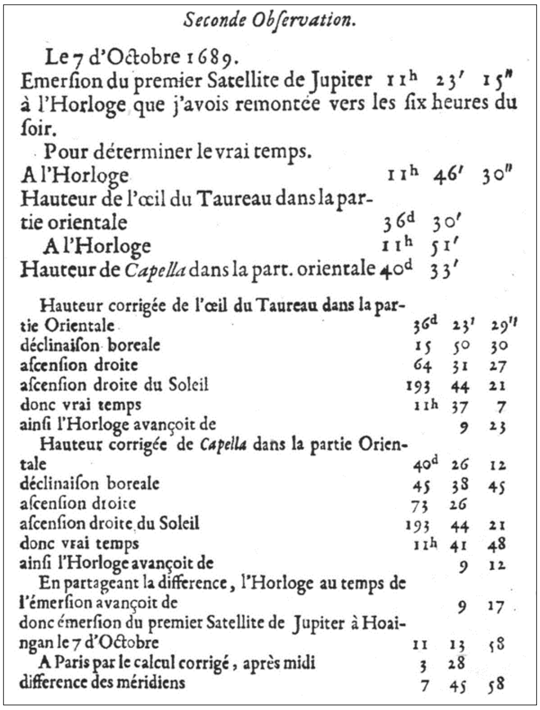
$$\Delta h = \Delta\delta / (\tan \xi \cos \delta) \qquad (5)$$

If the change in declination is positive, the correction to the hour angle will be applied negative and vice versa. The correction Δh is small, less than one minute.

If the morning and the corrected afternoon timings are T_1 and T_2 respectively, true noon according to the uncorrected clock is given by $T = T_1 + (T_2 - T_1) / 2$, and the clock correction is 12 hours – T.

4 PRACTICAL CALCULATION

An example of a clock error calculation using the first method is shown in Figure 3. It was made in preparation for the emersion/egress of Io just before midnight as observed from the Chinese town of Hoai-nghan (present day Huai'an in the province of Jiangsu) on 7 October 1689. The stars observed were α Tauri Aldebaran) and α Aurigae (Capella). Both stars were east of the meridian. The uncorrected clock time of the measurement of the altitude of the first one was 11:46:30 (PM). The first item in the calculation is the altitude of the star corrected for refraction. Then follows the declination of the star, its right ascension, the right ascension of the Sun, interpolated from tables for the long-itude of Paris with a longitude time difference of seven hours. Then the true time is calculated and it is found that the clock is 9 minutes and 23 seconds fast. With a second measurement at 11:51:0 using Capella the calculation gives that the clock is fast by 9 minutes 12 seconds. The value used to correct the time of the emersion was the average, 9 minutes 17 sec-



Seconde Obſervation.

Le 7 d'Octobre 1689.
Emerſion du premier Satellite de Jupiter 11h 23′ 15″ à l'Horloge que j'avois remontée vers les ſix heures du ſoir.

Pour déterminer le vrai temps.

A l'Horloge	11h	46′	30″
Hauteur de l'œil du Taureau dans la partie orientale	36d	30′	
A l'Horloge	11h	51′	
Hauteur de *Capella* dans la part. orientale	40d	33′	
Hauteur corrigée de l'œil du Taureau dans la partie Orientale	36d	23′	29″
déclinaiſon boreale	15	50	30
aſcenſion droite	64	31	27
aſcenſion droite du Soleil	193	44	21
donc vrai temps	11h	37	7
ainſi l'Horloge avançoit de		9	23
Hauteur corrigée de *Capella* dans la partie Orientale	40d	26	12
déclinaiſon boreale	45	38	45
aſcenſion droite	73	26	
aſcenſion droite du Soleil	193	44	21
donc vrai temps	11h	41	48
ainſi l'Horloge avançoit de		9	12
En partageant la difference, l'Horloge au temps de l'émerſion avançoit de		9	17
donc émerſion du premier Satellite de Jupiter à Hoai-ngan le 7 d'Octobre	11	13	58
A Paris par le calcul corrigé, après midi	3	28	
difference des méridiens	7	45	58

Figure 3: The clock error calculation for Huai'an on 7 October 1689 (after Anonymous, 1729: 782).

onds. The average of seven similar pairs of timings finally gives a longitude of Huai'an of 118° 50′ E of Greenwich as compared to the modern value of 119° 1′ E.

Figure 4 shows the details of a correction calculation taken from Anonymous (1729) using the method with two equal altitudes of the Sun on 12 July 1689. The event prepared for was the immersion/ingress of the Galilean moon Io behind Jupiter on the following night, observed from the town of Sian Fu. The calculations were done with a precision of seconds, often with fractions of seconds. There

Obſervations pour vérifier l'Horloge.

Le 12 de Juillet, hauteurs du bord ſupérieur du Soleil.

Temps du matin.			*Hauteurs.*	*Temps du ſoir.*		
9^h	18′	25″ $\frac{1}{2}$	53^d	2^h	40′	9″ $\frac{1}{2}$
	23	17 $\frac{1}{2}$	54		35	18
	28	16	55		30	18 $\frac{1}{2}$

J'ai ſuppoſé, pour les calculs ſuivans, la latitude de Si-nghan-fu de 34^d 16′ 30″ & la difference de longitude entre ſon méridien & celui de Paris de 7 heures : la latitude de Canton de 23^d 8′ & la longitude la même que celle de Si-nghan-fu.

Le 12 de Juillet, temps du matin	9^h	18′	25″ $\frac{1}{2}$
Temps du ſoir	2	40	9 $\frac{1}{2}$
Difference	5	21	44
Moitié de la difference	2	40	52
Diſtance du Soleil au méridien, à peu-près vrai	40^d	13′	0″
Hauteur du Soleil corrigée	52	59	6
Complément de la hauteur	37	0	54
Complément de la hauteur du Pole	55	43	30
Angle au Soleil	62	24	40
Difference de la déclinaiſon pour 24 heures	0	8	24
Déclinaiſon proportionnée à la difference des temps des obſervations	0	1	52
Augmentation ſuivant le parallelle du jour	0	0	8
Somme	0	2	0
Correction à ajouter au temps d'après-midi	0	1	2 $\frac{1}{2}$
Qui valent en parties de temps	0	0	4
Temps du ſoir corrigé	2^h	40′	13″ $\frac{1}{2}$
Difference entre le temps du matin & le temps du ſoir corrigé	5	21	48 $\frac{1}{2}$
Moitié de la difference	2	40	54 $\frac{1}{4}$
Heures de l'horloge au vrai midi du Soleil	11	54′	19 $\frac{3}{4}$
Retardement de l'horloge	0	0	40 $\frac{1}{4}$
A Si-nghan-fu immerſion obſervée le 13 de Juillet à l'horloge non corrigée	2	36	15
Donc immerſion au vrai temps à	2	36	55 $\frac{1}{4}$

Figure 4: Details of a correction calculation for two equal altitudes of the Sun taken on 12 July 1689 (after Anonymous, 1729: 860–861).

is one typographical error: the number 54 on the next-to-bottom-line should be 59, but otherwise all numbers are correct. The calculation follows exactly Fonteney's scheme. It was found that the clock was slow by 40 seconds and that the corrected local apparent time of the immersion was 2 hours 36 minutes 55 seconds after midnight on 13 July. According to Cassini's ephemerides for the Jovian moons, the apparent local time of the immersion in Paris was 7 hours 31 minutes 0 seconds on the evening of 12 July. The time difference, 7 hours 5 minutes and 55 seconds, translates into a longitude difference of 106° 28′ 49″ east of Paris. With a modern longitude of 2° 20′ 14″ E of the Paris Observatory this gives a longitude of 108° 49′ 3″ E from Greenwich, not far from the correct modern value of 108° 58′ E. The arc second precision is illusory.

For 14 September 1689 there is a calculation of the true time of the emersion of the Jovian moon Io as observed from Huai'an (Anonymous, 1729: 779). However, the observer (Father François Noël) notes that the longitude difference with Paris that he derives deviates from what several other observers had obtained earlier, and he concludes that the observed moon must have been one of the other satellites of Jupiter. Indeed, it can be shown that the satellite he observed was Ganymede.

5 CONCLUDING REMARKS

Anonymous (1729) reports only one example using the first method and the altitude of the Sun. There are 14 calculations using the altitudes of bright stars: α Lyrae (Vega), α Tauri (Aldebaran), α Aquilae (Altair), α Aurigae (Capella), α Orionis (Betelgeuse), α Canis Major (Sirius), and β Orionis (Rigel). For each set of timings a selection of two stars was used. There are six measurements on different dates using timings of equal altitudes of the Sun before and after noon, each of them having three pair of timings with slightly different altitudes.

The calculations to establish the clock corrections are, especially with the second method, quite time-consuming and difficult to do by hand using the tools available at the seventeenth century and are also prone to errors of calculation. However, checking the calculations with a modern computer shows that the Jesuits were extremely good at their work. In the more than thirty records in Anonymous (1729) there are a few typographical errors, four of the calculations have a final error of more than 6 seconds, and none has an error greater than 8 seconds. The average calculation error is a little more than one second. In the timings of the events for finding the longitude, the clock correction would be determined close to or before and after the event and the used clock error at the time of the event was an interpolated value of these corrections.

Using the derived event timings, the Jesuits were able to determine the longitudes of several locations in China with an error of the order of 10 arc minutes (Gislén, 2017). The French Jesuits were greatly favoured by being able to make their measurements on *terra firma*, which enabled them to achieve a high precision in their measurements.

Determination of the longitude at sea on an unstable ship platform is another story and was a big problem far into the eighteenth century, with inexact longitude positions causing several serious disasters at sea (de Grijs, 2017). The English Parliament passed an act in 1714 with a reward of £20,000 for any person who could find a method to determine the longitude at sea with an accuracy better than one-half of a degree. It was the English carpenter and clock-maker John Harrison (1693–1776) who finally succeeded in constructing a sufficiently accurate chronometer and eventually received about half the prize sum (Andrews, 1996).

6 NOTES

1. It is not known exactly what tables the Jesuit brough with them on their missionbut we can be assured that they were the most modern French ones at the time with Paris Observatory then headed by the renowned astronomer Giovanni Cassini.

The tables needed would have been tables of right ascension and declinations of stars and as well as tables (or means to compute) of the daily declinations of the Sun as well as time tables for the ingresses and egresses of the Galilean satellites for the longitude of Paris.

At that time, declination and right ascensions were known with an error less than one arc minute, the declination of the Sun even better.

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