AN ANALYSIS OF THE GOLDEN NUMBERS ON THE CALENDAR DISK OF THE ASTRONOMICAL CLOCK IN LUND

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Abstract: We investigate the golden numbers on the calendar disk of the astronomical clock in Lund, Sweden. Using methods of internal consistency we find printing errors and locate several golden numbers misplaced by one day. However, from the available data it is not possible to determine the principles that were used for calculation of the location of these golden numbers when the calendar disk was restored in 1923.

Keywords: Astronomical clock, Lund, golden numbers.

1 INTRODUCTION

The astronomical clock in the cathedral in Lund was originally constructed about CE 1420 as one of the several magnificent Hanseatic clocks in the Baltic region (Gislén, 2020a; Mogensen, 2008; Schukowski, 2006). Around the seventeenth century it had stopped working and in the subsequent centuries it was neglected and finally most parts of it ended up being stored in the attic of the cathedral; only the clock dial was preserved and displayed on the southeast wall of the church. In the beginning of 1900, there was a decision to restore the clock initiated by the Swedish architect Theodor Wåhlin (1923). The missing calendar disk of the old clock had to be completely reconstructed using ideas of layout from other preserved Hanseatic clocks from the same time in Europe. The renovated clock was inaugurated in 1923.

2 THE CALENDAR DISK

The calendar disk of the clock (Figure 1) has a diameter of 2.36 meters. The periphery of the disk is divided into the twelve months of the year. The next section is divided into 366 days of the year, including the leap day that is skipped by the clock mechanism in common years. Then follows the Sunday letter sequence A, B, C, D, E, F, G, starting with A on 1 January and repeating through the year and ending with A on 31 December. If you know the Sunday letter of the year, every date marked with that letter will be a Sunday. The Sunday letter is used to determine the date of Easter Sunday. The next section of the calendar disk gives the Swedish name attached to that date. Then follows a section with the golden numbers that are used to determine the date of New Moons of a particular year. Inside this section are the dates expressed in the complicated old Roman calendar. The central part of the calendar disk displays, for each year from 1923 to 2123, the Sunday letter, the golden number, the epact (the age of the Moon on 1 January) and the date of Easter Sunday.

3 THE GOLDEN NUMBERS

Golden numbers are used to predict New Moons. Nineteen tropical solar years are almost precisely equal to 235 synodic months, which means that New Moons on average repeat on the same date with an interval of nineteen years. This is the Metonic Cycle, after Meton, a Greek astronomer who lived in the fourth century before the Christian Era. Thus, we can assign a number, the golden number, from one to nineteen to every year that will characterise the New Moon dates of that year. The golden number of a particular year is easy to calculate: you divide the year by nineteen, keep the remainder, and add one. For example, the golden number of 2020 is 7.

The calendar disk of the astronomical clock in Lund has a set of such golden numbers written with red Roman letters, see Figure 1. New Moons are expected to occur, on average, on the dates that are marked with the golden number of the specific year. The golden numbers on the dates follow a certain sequence. Starting in January the series is 1, 9, 17, 6, 14, 3, 11, 19, 8, 16, 5, 13, 2, 10, 18, 7, 15, 4, and 12 and it is then repeated throughout the year. Mathematically the series can be generated by adding 8, modulo 19 to the previous number.

4 THE ANALYSIS

What started my interest in the golden numbers on the calendar disk was the discovery that there were a number of errors in the sequence given above. Some of the golden numbers were wrongly printed at the restoration of the clock in 1923, obviously by some Roman letters being missed. Thus, we have

- 10 February, X, should be XIX
- 26 April, V, should be IV
- 29 July, XII, should be XVII
- 15 October, XVII, should be XVIII

Figure 1: Part of the calendar disk showing the month of November (photograph: Lars Gislén).

- 6 November, VI, should be XVI
- 23 November, X, should be IX
- 24 November, VII, should be XVII

At the restoration, the golden numbers were painted on the calendar disk using stencils, presumably one at the time, first of all the 'I's, then the 'V's and finally the 'X's. It would then have been possible to miss one or more of the Roman letters out of 235 golden numbers on the disk.

Assuming that we have corrected these obvious errors the next question is: What principle was used for locating the golden numbers at specific dates on the calendar disk? The calendar disk was designed to cover the years CE 1923–2123 and a method was needed that would give usable golden numbers in order to predict the New Moons for this period.

According to the preface of Theodor Wåhlin's *Horologium Mirabile Lundensis* (Wåhlin, 1923), the calendrical computations were made by the son of the clockmaker Julius Bertram-Larsen who led the practical reconstruction of the clock. The calculations were checked by C.V.L. Charlier, then Professor at the Astronomical Department at the University of Lund. As far as I know there are no remaining documents that show how these calculations were made. The astronomical clock in Gdansk is the only other Hanseatic clock remaining that has a calendar disk with a set of golden numbers attached to the dates. However, that clock gives dates and times for the mean New Moons during four consecutive periods of 19 years: CE 1463–1538 (Gislén, 2020a).

A possible method for the reconstruction would have been the ecclesial computational method (Computus) that is used, combined with the Sunday letter, to calculate the date of the Easter Sunday. The methods in Computus assign golden numbers to the days of the year. A comparison between the Lund clock and Computus is shown in Table 1 below for the months March and April. Column G shows the golden numbers from Computus, G1 those of the clock. It is obvious that there is no match. The deviations are substantial and in general the golden numbers on the disk are located earlier than those of Computus. This can be explained by that the astronomical New Moon occurs when the Moon and the Sun have the same ecliptic longitude while the first emerging crescent of the New Moon, by tradition used to determine the first day in the ecclesial lunar calendar in Computus, cannot be seen until one or two days

Table 1: Comparison between Computus and clock golden numbers for March and April.

	Date	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
March	G	12	1		9		17	6		14	3		1		19	8		16	5		13	2		10		18	7		15	4		12
	G1	1	9		17	6		4		3	11		19	8		16	5		13	2		10		18	7		15	4	12		1	
April	G	1		9	17	6		14	3		1		19	8		16	5		13	2		10		18	7		15	4		12	1	
	G1	9	17		6		14	3		11	19		8	16		5	13		2		10	18		7	15		4	12		1	9	

after the astronomical New Moon. It is very probable that the golden numbers of the calendar disk are meant to show the average astronomical New Moon. This would also make it agree with the Moon phase and Moon pointer on the clock dial which displays the relative position of the Moon and the Sun.

It is unlikely that the golden numbers would mark the day of the true New Moon. The true Moon does not move with constant speed around the Earth, primarily because its orbit is not circular, partly because it has an inclination relative to the ecliptic, and also because the Moon is disturbed by gravitational forces from the Sun. This determines that the true New Moons can deviate substantially from the mean New Moons in a rather irregular way. Therefore, we can assume that the golden numbers of the clock were based on mean New Moons. The dates of the mean New Moons can be calculated given a start date and time for a New Moon by successively adding a synodic month of 29.530588 days, to give the precise times and dates for the subsequent New Moons. At the time of the reconstruction of the calendar disk astronomers in Lund also had access for instance to the calendrical tables by Schram (1908) which could be used to calculate mean New Moons. These tables can be checked by modern astronomical algorithms (Meeus, 1998) and shown to give essentially the same result, the difference being only about ten minutes.

One problem in assigning a golden number to a date is that a 19-year cycle of Julian or Gregorian years can contain either four or five leap days. This means that the assigned calendar date can vary with plus or minus one day between different 19-year cycles. A possibility is then to use a cycle of four 19-year cycles (76 years) that will contain a fixed number of leap days and calculate the average date. However, there is also another problem. Neglecting the year CE 2100 we can count using Julian solar years with on average 365.25 days. A 76-year period then contains 27 759 days precisely. The number of synodic months in such a cycle is 4·235 = 940. As each synodic month has 29.530588 days the period contains in total 940·29.530588 = 27 758.75 days. The date of the New Moon thus moves about 0.25 days back in time in 76 years. A possible solution is to use a data base (Gislén, 2020b) and compute averages using the 2 486 mean New Moons dates and times in for the entire period of the calendar disk, CE 1923 to CE 2123. Times in the data base were corrected for the Central European Time (CET) used in Lund. This solution is shown in Table 2. The computer gives the average date and time of day for each of each the 235 New Moons in a 19-year cycle. There is a question of how to round that time to a determine a definite date. I chose the previous day if the time was before noon, as this gives a rather satisfactory and in fact quite optimal fit to the actual location of the golden numbers on the calendar disk. This does not mean that the reconstruction used this averaging method but gives a way to compare the golden numbers on the calendar disk with a kind of standard. Also, from a practical point of view such a calculation by hand would not have been feasible at the time of the reconstruction considering the volume of computations involved, some kind of shortcut must have been made. However, in about 14% of the New Moons there is an annoying major deviation of one day.

It is possible to check the internal consistency of the golden numbers of the calendar disk in several ways without reference to the standard. The mean New Moons of a year should progress by on average about 29.5 days from lunation to lunation. In practice, we would expect steps of either 29 or 30 days. Placing the golden numbers of the disk in a matrix of 30·12 + 5 days, it is possible to follow this progression, Table 3. The months are indicated by the alternating colours. If the progression is 30 days the golden numbers will stay on the same row, with 29 days they will move up one row. In general this is what we see but there is at least one glaring anomaly: golden number 12 at 28 February is located an impossible 28 days from the corresponding new month in March, indicating an error in the location of this golden number on the disk.

A second way to check the internal consistency is to use the difference between a lunar year of twelve synodic months and a solar year. Twelve synodic months of 29.53059 days give a lunar year of 354.37 days. A solar year has on average about 365.25 days. The difference is 10.88 days which means that a New Moon each year on average steps back this number

Table 2: The columns marked with 'G' show the golden numbers. For each month the mean computed date is given, in the next column rounded to an integer day and then compared with the disk number.

G	Jan			Feb			Mar			Apr			Maj			Jun			G
1	1.61	1	1				1.58	1	1										1
9	2,14	2	2	1.67	1	1	3.02	2	2	1.61	1	1							9
17	4.74	4	4	3.27	2	2	4.61	4	4	3.14	2	2	2.67	2	2				17
6	6.33	5	5	4.86	4	4	6.12	5	5	4.65	4	4	4.18	3	3	2.71	2	2	6
14	7.95	7	7	6.48	5	6	7.74	7	7	6.27	5	6	5.80	5	5	4.33	3	3	14
3	9.44	8	9	7.97	7	7	9.20	8	9	7.73	7	7	7.26	6	7	5.79	5	5	3
11	10.95	10	10	9.48	8	9	10.83	10	10	9.36	8	9	8.89	8	8	7.42	6	7	11
19	12.51	12	11	11.15	10	10	12.27	11	12	10.80	10	10	10.33	9	10	8.86	8	8	19
8	14.05	13	13	12.58	12	12	13.84	13	13	12.37	11	12	11.90	11	12	10.43	9	10	8
16	15.58	15	15	14.11	13	14	15.44	14	15	13.97	13	13	13.50	13	13	12.03	11	11	16
5	17.14	16	16	15.67	15	15	17.02	16	16	15.55	15	15	15.08	14	14	13.61	13	12	5
13	18.76	18	18	17.29	16	16	18.64	18	18	17.17	16	16	16.60	16	15	15.23	14	14	13
2	20.27	19	19	18.80	18	18	20.13	19	19	18.67	18	18	18.20	17	17	16.73	16	16	2
10	21.85	21	21	20.39	19	20	21.64	21	21	20.18	19	20	19.71	19	19	18.24	17	18	10
18	23.44	22	23	29.97	21	21	23.20	22	23	21.73	21	21	21.26	20	21	19.79	19	19	18
7	24.95	24	24	23.48	22	23	24.74	24	24	23.27	22	23	22.80	22	22	21.33	20	20	7
15	26.57	26	26	25.10	24	24	26.36	25	26	24.89	24	24	24.42	23	24	22.95	22	22	15
4	28.04	27	27	25.57	26	26	27.80	27	27	26.33	26	26	25.86	25	25	24.39	23	24	4
12	29.67	29	29	28.20	27	28	29.46	28	28	27.99	27	27	27.52	27	27	26.05	25	25	12
1	31.14	30	31				31.02	30	30	29.55	29	29	29.08	28	28	27.61	27	27	1
9										31.08	30	30	30.61	30	30	29.14	28	28	9
17													32.20	31	31	30.73	30	30	17
6																			6
14																			14
3																			3
11																			11

G	Jul			Aug			Sep			Okt			Nov			Dec			G
1																			1
9																			9
17																			17
6	2.24	1	1																6
14	3.86	3	3	2.39	1	2													14
3	5.32	4	5	3.86	3	3	2.39	1	2	1.92	1	1							3
11	6.95	6	6	5.48	4	5	4.01	3	3	3.54	3	2	2.08	1	1	1.61	1	30	11
19	8.39	7	8	6.92	6	6	5.45	4	4	4.98	4	4	3.51	3	3	3.05	2	2	19
8	9.96	9	9	8.49	7	7	7.02	6	6	6.55	6	5	5.08	4	4	4.61	4	4	8
16	11.56	11	10	10.09	9	9	8.62	8	7	8.15	7	7	6.68	6	6	6.21	5	5	16
5	13.11	12	12	11.68	11	10	10.21	9	9	9.74	9	9	8.27	7	7	7.80	7	7	5
13	14.76	14	13	13.29	12	12	11.83	11	11	11.36	10	10	9.89	9	9	9.41	8	8	13
2	16.26	15	15	14.79	14	14	13.32	12	12	12.85	12	12	11.38	10	11	10.91	10	10	2
10	17.77	17	17	16.30	15	15	14.83	14	13	14.36	13	13	13.17	12	12	12.42	11	12	10
18	19.33	18	18	17.86	17	17	16.39	15	16	15.92	15	15	14.45	13	14	13.98	13	13	18
7	20.85	20	20	19.39	18	19	17.92	17	17	17.46	16	16	15.99	15	15	15.52	15	15	7
15	22.48	21	22	21.01	20	20	19.54	19	18	19.07	18	18	17.61	17	17	17.14	16	16	15
4	23.92	23	23	22.45	21	21	20.98	20	20	20.51	20	20	19.04	18	18	18.57	18	18	4
12	25.58	25	25	24.11	23	23	22.64	22	22	22.17	21	21	20.70	20	20	20.23	19	19	12
1	27.15	26	26	25.68	25	25	24.21	23	23	23.74	23	23	22.27	21	21	21.80	21	21	1
9	28.67	28	28	27.20	26	26	25.73	25	25	25.27	24	24	23.80	23	23	23.33	22	22	9
17	30.26	29	29	28.79	28	28	27.32	26	26	26.85	26	26	25.38	24	24	24.91	24	24	17
6	31.77	31	31	30.30	29	29	28.83	28	28	28,36	27	27	26.89	26	26	26.42	25	26	6
14				31.92	31	31	30.45	29	29	29.98	29	29	28.51	28	28	28.04	27	27	14
3										31.45	30	31	29.98	29	29	29.51	29	29	3
11																31.05	30	30	11

of days in the calendar. Counting with integer days it will be 11 days most of the time and 10 days now and then. Thus, if we insert the golden numbers of the year from the disk in a matrix with 11 rows, we expect that the golden numbers in a row would decrease by one unit as we move to the right. Now and then we expect that the golden number step up a row.

This is also what we see if we insert the computer-generated golden numbers (Table 4a). Table 4b shows the matrix with the golden numbers from the disk. Many of the golden numbers there behave as expected but there are several anomalies always also associated with deviations from the standard and indicate an error.

One example is on row 4 with the sequence of golden numbers 17, 16 14, 14, 12, 11, 10. The golden number 13 (15 May) would fit nicely into the sequence and fill the gap if it was moved one day forward. A problem with many of the golden numbers in the matrix is that they imply a difference 12 days to the next number. This is not totally impossible but should be extremely rare though it is quite common in the matrix. An obvious example is golden number 13 in column 13 that begs to be moved down one day. In some of the cases there are several possibilities to eliminate 12-day steps. In row 3, column 5 and row 4, column 6 you can either move golden number 13 down or move golden number 12 up or move both. But no matter what method has been used for the golden numbers, the conclusion must be that many of them are located one day wrong, most of them one day too early.

Table 3: 30-day progression

	1	2	3	4	5	6	7	8	9	10	11	12	
1	1	1	9	9		17	17		6	6			14
2	9	9		17	17		6	6		14	14	14	
3		17	17		6	6			14			3	3
4	17		6	6		14	14	14		3	3	11	11
5	6	6			14			3	3	11	11		
6			14	14		3	3		11			19	
7	14	14		3	3		11	11	19	19	19		
8		3	3		11	11		19		8	8	8	
9	3		11	11		19	19	8	8			16	
10	11	11		19	19		8		16	16	16		
11	19	19	19			8	16	16			5	5	
12			8	8	8	16		5	5	5		13	
13	8	8		16	16	5	5			13	13		
14			16		5		13	13	13			2	
15	16	16	5	5	13	13			2	2	2		
16	5	5		13			2	2	10	10	10	10	
17		13	13		2	2		10				18	
18	13		2	2			10			18	18		
19	2	2			10	10	18	18	18	7	7	7	
20			10	10		18			7			15	
21	10	10		18	18	7	7	7	15	15	15		
22		18	18		7			15			4	4	
23	18		7	7		15	15	4	4	4		12	
24	7	7		15	15		4			12	12		
25		15	15		4	4		12	12		1	1	
26	15		4	4		12	12		1	1		9	
27	4	4	12	12	12		1	1		9	9		
28					1	1		9	9		17	17	
29	12	(12)	1	1		9	9		17	17			
30		1		9	9		17	17		6	6	6	

Table 4a: Matrix with 11 rows of the golden numbers of the standard.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
1	1	19					3	2	1	19	18	17	16					19	18	17	16	15	14	13	12	11						13	12	11
2	9	8	7	6	5	4				8	7	6	5	4	3	2	1					4	3	2	1	19	18	17	16	15	14			
3				14	13	12	11	10	9						11	10	9	8	7	6	5						7	6	5	4	3	2	1	
4	17	16	15				19	18	17	16	15	14	13	12					15	14	13	12	11	10	9	8						10	9	
5	6	5	4	3	2	1						3	2	1	19	18	17	16					19	18	17	16	15	14	13	12	11			
6				11	10	9	8	7	6	5	4				8	7	6	5	4	3	2	1						3	2	1	19	18	17	
7	14	13	12				16	15	14	13	12	11	10	9						11	10	9	8	7	6	5	4						6	
8	3	2	1	19	18	17						19	18	17	16	15	14	13	12						14	13	12	11	10	9	8	7		
9					7	6	5	4	3	2	1					4	3	2	1	19	18	17	16	15					18	17	16	15	14	
10	11	10	9	8				12	11	10	9	8	7	6	5					8	7	6	5	4	3	2	1	19						
11		18	17	16	15	14	13						15	14	13	12	11	10	9							10	9	8	7	6	5	4	3	

Table 4b: Matrix with 11 rows of the golden numbers on the disk. Red numbers denote major deviations from the standard.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
1	1		18		16			2	1	19	18	17	16	15					18	17	16	15	14	13	12							13	12	11
2	9	8	7	6	5	4	3					6	5	4	3	2	1	19			5	4		2	1	19	18	17	16	15	14			
3					13		11	10	9	8	7		13				9	8	7	6			3	10		8	7	6	5	4	3	2	1	
4	17	16	15	14		12			17	16	15	14		12	11	10		16			13	12	11		9								9	
5	6	5	4	3	2	1	19	18					2	1	19	18	17		15	14			19		17	16	15	14	13	12	11	10		
6						9	8	7	6	5	4	3				7	6	5	4	3	2	1		18						1	19	18	17	
7	14	13	12	11	10					13	12	11	10	9	8			13			10	9	8	7	6	5	4	3	2					
8		2		19	18	17	16	15	14					17	16	15	14		12	11			16	15	14	13	12	11	10	9	8	7	6	
9	3		1			6	5	4	3	2	1	19	18		5			2	1	19	18	17								17	16	15	14	
10	11	10	9	8	7			12			9		7	6		4	3			8		6	5	4	3	2	1	19	18					
11	19		17		15	14	13		11	10		8		14	13	12	11	10	9		7				11	10	9	8	7	6	5	4	3	

5 CONCLUSION

It seems probable that some kind of averaging method was used for the calculation of the location of the golden numbers on the calendar present it is not possible to determine precisely which method. However, by looking into the internal consistency of the data available it is possible to localise a number of very likely one-day errors. Here we give a tentative list of such errors:

- January: Golden number 18 and 1 (31 January)
- February: Golden number 12
- March: Golden number 3 and 12
- May: Golden number 8 and 13
- June: Golden number 5
- July: Golden number 13 and 16
- August: Golden number 5 and 7
- September: Golden number 9, 10, and 18 or 19

As the true Moon can deviate from the mean Moon by about one day, the impact of these errors for the average person using the calendar disk as a means to predict the New Moon would, with exception of the printing errors, hardly be noticed.

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Professor Lars Gislén was born in Lund (Sweden) in 1938, and received a PhD in high energy particle physics from the University of Lund in 1972. He worked in 1970/1971 as a re-searcher at the Laboratoire de physique théorique in Orsay (France) with models of high energy particle scattering. He has also done research on atmospheric optics and with physical modelling of biological systems and evolution.

He has worked as an Assistant Professor (University Lector) at the Department of Theoretical Physics at the University of Lund, where he gave courses on classical mechanics, electrodynamics, statistical mechanics, relativity theory, particle physics, cosmology, solid state physics and system theory.

For more than twenty years he was a delegation leader and mentor for the Swedish team in the International Physics Olympiad and the International Young Physicists' Tournament.

Lars retired in 2003, and since then his interests have focused on medieval European astronomy and on the astronomy and calendars of India and South-east Asia. He has published more than 20 research papers in this field. He has also made public several spreadsheet tools implementing a number of astronomical models from Ptolemy to Kepler as well as computer tools for the calendars of India and South-east Asia. He is a member of the IAU.