Abstract: For finding the true positions of the Sun, the Moon and the five planets the Indian classical astronomical texts use the concept of the _manda_ epicycle which accounts for the equation of the centre. In addition, in the case of the five planets (Mercury, Venus, Mars, Jupiter and Saturn) another equation called _śīghraphala_ and the corresponding _śīghra_ epicycle are adopted. This correction corresponds to the transformation of the true heliocentric longitude to the true geocentric longitude in modern astronomy. In some of the popularly used handbooks (_karaṇa_) instead of giving the mathematical expressions for the above said equations, their discrete numerical values, at intervals of 15°, are given.

In the present paper using the data of discrete numerical values we build up continuous functions of periodic terms for the _manda_ and _śīghra_ equations. Further, we obtain the critical points and the maximum values for these two equations.

Keywords: Equation of the centre, epicycle, periphery, apogee, perigee, equation of the conjunction, _śīghraphala_, _mandaphala_, _paridhi_, _Grahalāghava_, Ganeśa Daivajña

1 INTRODUCTION

The _Grahalāghava_ (GL) is one of the most popular _karaṇa_ texts of Indian astronomy, and was written by the famous sixteenth-century author Ganeśa Daivajña. After Bhāskara-II of the twelfth century there was a decline for a brief period in the development of mathematics and astronomy in India. But we see tremendous work was done in the south i.e., in Kerala and Maharashtra, giving rise to some of the great and eminent luminaries like Nilakaṇṭa Somayājī and Ganeśa Daivajña.

Ganeśa Daivajña is unique because he dispensed with trigonometric terms in his computations and replaced them with suitable algebraic approximations. This method helped many almanac (_paṇcāṅga_) makers to do calculations in a simple way. So even today, the GL is one of the popular texts among almanac-makers.

The text of the GL consists of 187 verses (_slokas_) distributed in 14 chapters. In chapters 2 and 3 the true positions of the Sun, the Moon and the five planets are discussed. For the Sun and the Moon there is only one correction, namely the _mandaphala_, which corresponds to the equation of the centre, taking into account the eccentricity of the body’s orbit. But for the five planets, apart from the _mandaphala_ one more equation called _śīghraphala_ is applied. _Śīghraphala_ converts heliocentric position to geocentric position of the planets. In order to determine the two equations _manda_ and _śīghra_, Ganeśa Daivajña gives discrete values, called _mandāṅkas_ and _śīghrāṅkas_. These are obtained by multiplying the actual _manda_ and _śīghra_ corrections by 10. Further, these values are in arc minutes (_kalās_), and given in integers for every 15°. Ganeśa Daivajña does not provide either the peripheries (_paridhis_) of the epicycles nor does he mentions explicitly the expressions for the two equations. However, in the case of the Sun and the Moon he gives explicit approximate algebraic expressions for the equation of the centre. In this paper we estimate the ranges of peripheries of the equations for each of the bodies.

2 THE METHOD OF THE _GRAHALĀGHAVA_ FOR THE EQUATION OF THE CENTRE

In obtaining the mean positions of the Sun and the Moon it was earlier assumed that these bodies moved in circular orbits around the Earth with uniform angular velocities. However, observations revealed that the motions were non-uniform. The true positions were related to the epicyclic theory that is explained in the following section.
2.1 Epicyclic Theory and the Equation of the Centre

The theory is that while the mean Sun or the Moon move along a big circular orbit (see Figure 1), the actual Sun and Moon move along a smaller circle called an epicycle, whose centre is on the larger circle.

The larger circle ABP with the Earth E as its centre is called the deferent circle (kaksāvrta). Let A be the position of the mean Sun when the true Sun is farthest from the Earth. The line AEP is called the apse line and AE is the radius (trijyā) of this orbit. The epicycle, with A as centre and a prescribed radius (smaller than AE) is called the nucoccovṛtta. Let the apse line PEA cut the epicycle at U and N. The two points U and N are respectively called the apogee (mandocca) and the perigee (mandanīca) of the Sun. Note that as the Sun moves (as seen from Earth) along the epicycle, the Sun is farthest from the Earth at U and nearest at N.

The epicyclic theory assumes that as the centre of the epicycle (i.e. mean Sun) moves along the circle ABP in the direction of the signs of the zodiac (from west to east) with the velocity of the mean Sun, the true Sun itself moves along the epicycle with the same velocity but in the opposite direction (from east to west). Further, the time taken by the Sun to complete one revolution along the epicycle is the same as that taken by the mean Sun to complete a revolution around the orbit.

Now in Figure 1, suppose the mean Sun moves from A to A’. Let A’ and E be joined, cutting the epicycle at U’ and N’, which are the current positions of the apogee (mandocca) and the perigee (mandanīca). While the mean Sun is at A’, suppose the true Sun is at S on the epicycle so that U’A’S = U’EÂ. Join ES, cutting the orbit (i.e., circle ABP) at S’. Then A’ is the mean Sun (madhya Ravi) and S’ is the true Sun (spaṣṭa or sphaṭa Ravi). The difference between the two positions viz., A’ES’ (or arc A’S’) is called the equation of the centre (mandaphala).

In order to obtain the true position of the Sun, it is necessary to get an expression for the equation of the centre which will have to be applied to the mean position.

In Figure 1 SC and A’D are drawn perpendicular to U’N’E and U’N respectively. The arc AA’ (or AÅA’), the angle between the mean Sun and the apogee, is called the mean anomaly (mandakendra, henceforth MK) of the Sun.

We have, in the right-angled triangle A’DE, 
\[ \sin A'Å = \sin DÅA' = A'D/A'Å \]
so that, A’D = RsinAA’ = RsinMK is called R sine of anomaly (mandakendrajyā), where R = A’E and MK = arc AA’.

From the similar right-angled triangles SCA’ and A’DE, we have
\[ SC/SA' = A'D/AE' \]
and
\[ SC = (SA' \times A'D)/A'Å \]

Since SA’ and A’Å are respectively the radii of the epicycle and the orbit, these are proportional to the circumferences of the two circles; that is
\[ SA'/A'Å = \text{circumference of the epicycle/circumference of the orbit} \]
\[ \therefore SC = (\text{circumference of the epicycle} \times \text{mandakendrajyā})/360° \]

Now, taking SC approximately the same as A’S’, the equation of the centre (mandaphala, henceforth MPH) is given by
\[ R\sin (MPH) = \text{circumference of the epicycle} \times \text{mandakendrajyā})/360° \]
\[ = (p/R) \times R\sin MK \]
i.e. \( \sin (MPH) = (p/R) \times \sin MK \)
where RsinMK is the ‘Indian sine’ (jyā) of the anomaly MK of the Sun. The maximum value of the equation of the centre, i.e., \( \sin (MPH) \) is \( p/R \) (in radians) or \( p/2\pi \) (in degrees).

In his Grahalāghava, Ganeśa Daivajña gives the following verse to obtain the anomaly from the apogee (mandakendra) of the planet:

If the bhūja (of the manda anomaly) is less than three rāśis (signs) then take that itself, if the anomaly is greater than three rāśis and less than six rāśis then consider the difference of six rāśis (180°) and the anomaly as the bhūja, if the anomaly is greater than six rāśis...
and less than nine rāṣis then subtract six rāṣis (180°) from the anomaly to get the bhuja and if the anomaly is greater than nine rāṣis and less than twelve rāṣis then the remainder of subtracting it from twelve rāṣis (360°) is the bhuja. (Grahalāghava, Ch-II, śloka -1; our English translation).

This means the anomaly from the apogee (mandakendra, MK = apogee (mandocca) of the planet – Mean planet. MK is expressed as an acute angle; to get this, we use the following procedure:

1. If 0° ≤ MK < 90° then MK itself is the argument (bhuja) i.e., bhuja = MK.
2. If 90° ≤ MK < 180° then bhuja = 180° – MK
3. If 180° ≤ MK < 360° then bhuja = MK – 180°
4. If 270° ≤ MK < 360° then bhuja = 360° – MK

According to the Grahalāghava, the apogees of the heavenly bodies are as shown in Table 1.

It is assumed that the apogee of the Moon varies, whereas those of the other bodies are fixed.

The method of finding the equation of the centre of the Sun is explained in the following verse:

The difference between the mandocca (apogee) and the mean planet is called (manda) kendra (anomaly). If the kendra is within six rāṣis from Meṣa or within six rāṣis from Tūlā, (correspondingly) the mandaphala (the equation of the centre) is positive or negative.

In the case of Ravi (Sun), divide the bhuja (of the mandakendra) by 9, subtract it from 20 and multiply the result by itself; (this is the numerator). Divide the numerator by the difference between 57 and one-ninth of the numerator. (Grahalāghava, Ch-II, śloka -2; our English translation).

This means, find the anomaly from the apogee (MK) of the Sun and express MK in terms of bhuja of MK as explained earlier. Denote bhuja of MK by BMK.

1. Subtract (BMK/9) from 20 and multiply this by (BMK/9).
2. Divide the result of (1) by 9.
3. Subtract the result of step (2) from 57.
4. Express the results of step (3) and step (1) in seconds of arc (vikalās) and divide the result of step (1) by that of step (3).

Then the result is the equation of the centre of the Sun.

i.e., The equation of the centre of the Sun = [20 – (BMK/9)] × (BMK/9) / [57 – {(20 – (BMK/9)) × (BMK/9) / 9}]

Note:

1. In devising the above equation the author dispenses with the trigonometric ratio sine.

(2) If the anomaly from the apogee is within 6 signs from Aries (Meṣa) (i.e., 0° ≤ MK < 180°) then the equation of the centre is additive.
(3) If the anomaly from the apogee is within 6 signs from Libra (Tūlā) (i.e., 180° ≤ MK < 360°) then the equation of the centre is subtractive.
(4) If the anomaly is 0° or 180° then the equation of the centre is zero.

2.2 Rationale for the Equation of the Centre of the Sun

Śrīpati Bhaṭṭa’s (ca. tenth century) expression for the R sine (jyā) of the anomaly is as follows:

Subtract the manda anomaly from 180 and multiply by itself; (this is the numerator). Divide the numerator by the difference between 10125 and one-fourth of the numerator. (Finally) thus obtained result is multiplied by 120 to get the jyā (R sine) of the manda anomaly of the Sun. (Śiddhānta-śekhara, Ch-III, śloka-17; our English translation).

This implies the anomaly from the apogee (MK) in degrees is subtracted from 180° and the remainder is multiplied by the same quantity (MK). Then the result is divided by its one-fourth, subtracted from 10125. This result is multiplied by twice sixty (i.e., by 120).

i.e. In symbols, R sine of anomaly = [[(180 – MK)MK × 120] / {10125 – [(180 – MK)/4 × MK}]

where MK stands for the bhuja of the anomaly i.e., R sine (MK) = [(180 – MK)MK × 480] / [405000 – (180 – MK)MK]

= {

[(180 – MK) / 9] [(MK/9) × 480]} / {

[405000/(9 × 9)] – [(180 – MK)/9](MK/9)}

(dividing by 9 × 9)

= {[(20 – (MK/9)(MK/9) × 480)] / {500 – [20 – (MK/9)])/(MK/9)]

(1)

The above derivation is based on the significant and unique formula of Bhāskara I (c. 629 CE);

i.e., sin θ = [4 (180° – θ) θ] / [405000 – (180° – θ) θ]

Now, according to the Grahalāghava the maximum equation of the centre (parama mandaphala) of the Sun

= (125°/57) ≈ 2° 11’ 34”.

Table 1: Apogee of the heavenly bodies.

<table>
<thead>
<tr>
<th>Body</th>
<th>Apogee</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sun</td>
<td>78°</td>
</tr>
<tr>
<td>Mars</td>
<td>120°</td>
</tr>
<tr>
<td>Mercury</td>
<td>210°</td>
</tr>
<tr>
<td>Jupiter</td>
<td>180°</td>
</tr>
<tr>
<td>Venus</td>
<td>90°</td>
</tr>
<tr>
<td>Saturn</td>
<td>240°</td>
</tr>
</tbody>
</table>
The equation of the centre of the Sun = (125°/57) × (mandakendrajyā)/120

\[ = \frac{125}{57} \times \frac{1}{120} \times [(20 - (MK/9)(MK/9) \times 480] / 500 - [20 - (MK/9)(MK/9)] \]

Using (1)

\[ = \left\{ \frac{(125 \times 57)}{6} \right\} [20 - (MK/9)(MK/9) \times 4] / \{500 - [20 - (MK/9)(MK/9)] \}

\[ = \left\{ \frac{(500 \times 57)}{6} \right\} [20 - (MK/9)(MK/9)] / \{500 - [20 - (MK/9)(MK/9)] \}

\[ = \left\{ [20 - (MK/9)(MK/9)] / \{500(57)/6) \right\} [20 - (MK/9)(MK/9)] \]

The exact formula for the equation of the centre of the Sun = sin⁻¹[(p/R)sin MK] where R = 360°, p is the periphery of the manda epicycle (in degrees) and MK is the Sun’s anomaly (from the apogee, mandocca).

Using this formula with the range of MK from 15° to 90° the Sun’s equation of the centre, MPH, and the periphery (paridhi) of the manda epicycle, p, are estimated and listed in Table 2.

In order to estimate the manda periphery of the Sun from 0° to 90°, we adopt the formula \( p = A + B \sin(MK) \). The related procedure is explained in later sections. The periphery of the Sun for MK = 0° is 14°.001 and for MK = 90° is 13°.692.

Similarly, the equation of the centre of the Moon is given in the following verse

In the case of Vidhu (Moon), one-sixth of the manda anomaly is subtracted from 30 and the remainder is multiplied by the same; (this is the numerator). This numerator is divided by the difference between 56 and one-twentieth of the numerator. This is Moon’s equation of the centre. (Grahalāghava, Ch-II, śloka -3; our English translation).

This can be expressed as the following formula:

Equation of the centre of the Moon = \[ [(30 - (MK/6))(MK/6)] / [56 - (30 - (MK/6)(MK/6))] \]

2.3 Rationale for the Equation of the Centre of the Moon

We have R sine of anomaly = [(180 – MK)MK × 480] / [40500 – (180 – MK)MK]

According to Śrīpati Bhaṭṭa, dividing the numerator and the denominator by 6, then

\[ R_{\text{sin(MK)}} = \left\{ \frac{180 \times (\text{MK}) \times (\text{MK})}{480} \right\} / \left\{ \frac{40500}{6} \times (\text{MK}) \times (\text{MK}) \right\} \]

\[ = \left\{ \frac{30 \times (\text{MK})}{(\text{MK})} \times 480 \right\} / \left\{ 120 \times [1125 - 30 - (\text{MK})] \right\} \]

According to the Grahalāghava the maximum equation of the centre of the Moon = 5°.

Equation of the centre of the Moon = (5 × R sine of anomaly) / 120

\[ = \left\{ 5 \times (\text{MK}) \times (\text{MK}) \times 480 \right\} / \left\{ 120 \times [1125 - 30 - (\text{MK})] \right\} \]

\[ = \left\{ (2400/120) \times (\text{MK}) \times (\text{MK}) \right\} / \left\{ 1125 - 30 - (\text{MK}) \right\} \]

\[ = \left\{ 2[30 - (\text{MK})/6)](\text{MK})/6] \right\} / \left\{ 1125 - 30 - (\text{MK})/6)](\text{MK})/6] \right\} \]

\[ = \left\{ 30 - (\text{MK})/6)]\text{MK})/6] \right\} / \left\{ (1125/20)[30 - (\text{MK})/6)]\text{MK})/6] \right\} \]

\[ = \left\{ 30 - (\text{MK})/6)]\text{MK})/6] \right\} / \left\{ 56.25 - (30 - \text{MK})/6)]\text{MK})/6] \right\} \]

We have 32°.075 for MK = 0° and MK = 90°. It is 31°.591.

3 EQUATION OF THE CENTRE OF THE PLANETS

In the case of the five planets in the GL, instead of providing direct expressions, Ganeśa Daivajña gives discrete numerical values for the equation of the centre (mandaphala) in degrees at intervals of 15° of the manda anomaly. He has multiplied the equation of the centre by 10 (to avoid fractions) and calls them as mandānkas, as given in Table 4.

In order to estimate the underlying manda per-
we have the equation of the centre given by equation (5)

\[ \sin(MPH) = \left(\frac{R}{p}\right)\sin(MK) \]

(3) \[ \Rightarrow p = \left(\frac{MPH \times R}{\sin(MK)}\right) \]

(4) \[ \sin(MPH) = \left(\frac{R}{p}\right)\sin(MK) \]

where \( p \) is periphery of the epicycle, \( MK \) is the manda anomaly and \( R \) is \( 2\pi \) radians or 360°.

As an example, based on equation (4) the manda periphery \( p \) of Mars is given in Table 5.

We find from Table 5 that the manda periphery increases from \( 70^\circ.40145 \) to \( 81^\circ.68142 \) as the manda anomaly \( MK \) increases from \( 15^\circ \) to \( 90^\circ \).

Note: The manda periphery for \( MK = 0 \) cannot be obtained from equation (4) since the denominator vanishes.

Now since \( p \) varies from \( 70^\circ.40145 \) to \( 81^\circ.68142 \), we express the periphery \( p \) for any given \( MK \) in the form

\[ p = A + B \sin(MK) \]

(6)

for which we have to determine the constant coefficients \( A \) and \( B \). Tentatively, for \( MK = 30^\circ \) and \( 90^\circ \), we get the respective linear equations as

\[ p = A + (B/2) \quad \text{and} \quad p = A + B \quad \text{(7a)} \]

Solving these equations, we obtain \( A = 61^\circ.5752 \) and \( B = 20^\circ.10622 \). (It is to be noted that we do not get the same values of \( A \) and \( B \) as above if we consider the other pairs of the linear equations.)

This means that for the above values of \( A \) and \( B \), periphery \( p \) varies from \( 66^\circ.77908 \) to \( 81^\circ.68142 \) as \( MK \) varies from \( 15^\circ \) to \( 90^\circ \) in the case of Mars. Similarly, estimating the manda peripheries for the other four planets namely, Mercury, Venus, Jupiter and Saturn, we get the values as shown in Table 6.

When \( MK = 0^\circ \), formula (6) becomes \( p = A \) hence the above table of manda peripheries can be now listed for \( MK = 0^\circ \) to \( 90^\circ \) by solving equations (7) by finding the \( A \) and \( B \) values.

Now, considering the actual expression for the equation of the centre given by equation (5) we have

\[ \sin(MPH) = \left(\frac{R}{p}\right)\sin(MK) \]

(7b)

Following the same procedure as for Mars in the case of the remaining four planets we get the manda peripheries as shown in Table 7.

From Table 8, we find that the manda periphery \( p \) increases as anomaly \( MK \) increases from \( 0^\circ \) to \( 90^\circ \) in the case of superior planets viz. Mars, Jupiter and Saturn. On the other hand, in the case of the two interior planets Mercury and Venus \( p \) decreases as \( MK \) increases from \( 0^\circ \) to \( 90^\circ \).

Table 5: Manda periphery of Mars in degrees.

<table>
<thead>
<tr>
<th>MK</th>
<th>MPH</th>
<th>Manda periphery (p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15°</td>
<td>2.9</td>
<td>70°.40145</td>
</tr>
<tr>
<td>30°</td>
<td>5.7</td>
<td>71°.62831</td>
</tr>
<tr>
<td>45°</td>
<td>8.5</td>
<td>75°.52901</td>
</tr>
<tr>
<td>60°</td>
<td>10.9</td>
<td>79°.08165</td>
</tr>
<tr>
<td>75°</td>
<td>12.4</td>
<td>80°.65991</td>
</tr>
<tr>
<td>90°</td>
<td>13</td>
<td>81°.68142</td>
</tr>
</tbody>
</table>

Table 6: The range of manda peripheries of other planets.

<table>
<thead>
<tr>
<th>Planet</th>
<th>Manda periphery (p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury</td>
<td>28°.20784 22°.61947</td>
</tr>
<tr>
<td>Jupiter</td>
<td>33°.01997 35°.81416</td>
</tr>
<tr>
<td>Venus</td>
<td>15°.944   09°.42478</td>
</tr>
<tr>
<td>Saturn</td>
<td>46°.25    58°.43363</td>
</tr>
</tbody>
</table>

Table 7: The range of manda peripheries of all the planets for \( MK = 0^\circ \) and \( 90^\circ \) (using equation 7a).

<table>
<thead>
<tr>
<th>Planet</th>
<th>Manda periphery (p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mars</td>
<td>61°.5752 81°.68142</td>
</tr>
<tr>
<td>Mercury</td>
<td>30°.15929 22°.61947</td>
</tr>
<tr>
<td>Venus</td>
<td>18°.22124 09°.42478</td>
</tr>
<tr>
<td>Jupiter</td>
<td>32°.04424 35°.81416</td>
</tr>
<tr>
<td>Saturn</td>
<td>42°.09733 58°.43363</td>
</tr>
</tbody>
</table>

Table 8: The range of manda peripheries of all the planets (using equation 7b).

Manda peripheries according to some Indian classical astronomical texts are listed in Table 9, together with our computations for comparison.

From Table 9, it is interesting to note that the same behaviour is seen in the Āryabhaṭīya also. In fact, even the ranges of variation of the manda periphery as estimated based on the GL are close to those of the Āryabhaṭīya. However, in
the case of the Sun and the Moon the peripheries vary as in the Sūryasiddhānta.

### 4 EQUATION OF THE CONJUNCTION OF THE PLANETS

Ganeśa Daivajña has provided śīghrāṅkas similarly as in the case of the equation of the centre (mandaphala) for the convenience of computation. Actual equations of the conjunction (śīghrāphalas) are obtained from these śīghrāṅkas dividing by 10. The discrete numerical values of śīghrāṅkas for the intervals of 15° degrees are listed in Table 10.

In order to determine the śīghra peripheries of different planets we adopt the following procedure:

Śīghrāphala \((SPH) = \sin^{-1}\left(\frac{[(p/360) \sin{(SK)}]}{\sqrt{[(p/360)^2 \pm 2(p/360)\cos{(SK)} + 1]}}\right)\) (8)

where \(p\) in the śīghra periphery, \(SPH\) is the śīghrāphala and \(SK\) is the anomaly of the conjunction (śīghrakendra).

Here \(SK\) is the anomaly of the conjunction (with the Sun) i.e., \(SK\) is the Mean Sun – Mean planet for the superior planets. In the case of Mercury and Venus, \(SK\) is the Mean planet – Mean Sun.

Let \((p/360) = r\)

\(SPH = \sin^{-1}\left(\frac{r \sin{(SK)}}{\sqrt{r^2 \pm 2r \cos{(SK)} + 1}}\right)\) or

<table>
<thead>
<tr>
<th>Bodies</th>
<th>Computed Values based on GL</th>
<th>The Aryabhaṭiya</th>
<th>The Sūryasiddhānta</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sun</td>
<td>13°.69 – 14°</td>
<td>13°.5</td>
<td>13°.66 – 14°</td>
</tr>
<tr>
<td>Moon</td>
<td>31°.59 – 32°.07</td>
<td>31°.5</td>
<td>31°.66 – 32°.07</td>
</tr>
<tr>
<td>Mars</td>
<td>62°.03 – 60°.98</td>
<td>63°.81°</td>
<td>72° – 75°</td>
</tr>
<tr>
<td>Mercury</td>
<td>30°.16 – 22°.60</td>
<td>31°.5 – 22°.5</td>
<td>28° – 30°</td>
</tr>
<tr>
<td>Jupiter</td>
<td>32°.07 – 35°.75</td>
<td>31°.5 – 36°.5</td>
<td>32° – 33°</td>
</tr>
<tr>
<td>Venus</td>
<td>18°.22 – 09°.42</td>
<td>18° – 9°</td>
<td>11° – 12°</td>
</tr>
<tr>
<td>Saturn</td>
<td>42°.09 – 58°.43</td>
<td>40°.5 – 58°.5</td>
<td>48° – 49°</td>
</tr>
</tbody>
</table>

\(\sin{(SPH)} = \{[r \sin{(SK)}]/\sqrt{[r^2 \pm 2r \cos{(SK)} + 1]}\}\) (10)

On squaring both the sides and simplifying equation (10) we get a following equation:

\(r^2\sin^2{(SPH)} + 2r \cos{(SK)}\sin^2{(SPH)} + \sin^2{(SPH)} - r^2 \sin^2{(SK)} = 0\)

\([\sin^2{(SPH)} - \sin^2{(SK)}])r^2 + 2\cos{(SK)}\sin^2{(SPH)}r + \sin^2{(SPH)} = 0\)

This equation is of the form \(Ar^2 + Br + C = 0\), which is a quadratic equation, where \(A = [\sin^2{(SPH)} - \sin^2{(SK)}], B = 2\cos{(SK)}\sin^2{(SPH)}\) and \(C = \sin^2{(SPH)}\).

The roots of a quadratic equation \(Ar^2 + Br + C = 0\) are:

\(r = \{-B \pm \sqrt{[B^2 - 4AC]} \}/2A\)

or

\(r = \{-B \mp \sqrt{[B^2 - 4AC]} \}/2A\)

Between these two roots, the valid solution is provided by the equation

\(r = \{-B \mp \sqrt{[B^2 - 4AC]} \}/2A\)

From equation (9) we have \(p = 360° \times r\).

Thus the śīghra periphery

\(p = 360° \times \{-B + \sqrt{[B^2 - 4AC]} \}/2A\) (11)

Using the above equations we computed the śīghra peripheries of Mars and listed the values in Table 11.

<table>
<thead>
<tr>
<th>Planets</th>
<th>15°</th>
<th>30°</th>
<th>45°</th>
<th>60°</th>
<th>75°</th>
<th>90°</th>
<th>105°</th>
<th>120°</th>
<th>135°</th>
<th>150°</th>
<th>165°</th>
<th>180°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mars</td>
<td>58°</td>
<td>117°</td>
<td>174°</td>
<td>228°</td>
<td>279°</td>
<td>325°</td>
<td>365°</td>
<td>393°</td>
<td>400°</td>
<td>368°</td>
<td>249°</td>
<td>0</td>
</tr>
<tr>
<td>Mercury</td>
<td>41°</td>
<td>81°</td>
<td>117°</td>
<td>150°</td>
<td>178°</td>
<td>199°</td>
<td>212°</td>
<td>212°</td>
<td>195°</td>
<td>155°</td>
<td>89°</td>
<td>0</td>
</tr>
<tr>
<td>Jupiter</td>
<td>25°</td>
<td>47°</td>
<td>68°</td>
<td>85°</td>
<td>98°</td>
<td>106°</td>
<td>108°</td>
<td>102°</td>
<td>89°</td>
<td>66°</td>
<td>36°</td>
<td>0</td>
</tr>
<tr>
<td>Venus</td>
<td>83°</td>
<td>126°</td>
<td>186°</td>
<td>246°</td>
<td>302°</td>
<td>354°</td>
<td>402°</td>
<td>440°</td>
<td>461°</td>
<td>443°</td>
<td>326°</td>
<td>0</td>
</tr>
<tr>
<td>Saturn</td>
<td>15°</td>
<td>28°</td>
<td>39°</td>
<td>48°</td>
<td>54°</td>
<td>57°</td>
<td>57°</td>
<td>53°</td>
<td>45°</td>
<td>33°</td>
<td>18°</td>
<td>0</td>
</tr>
</tbody>
</table>

From Table 11 as \(SK\) varies from 15° to 165° the śīghra periphery ‘\(p\)’ varies from 227.545 to 236.297. We express the śīghra periphery ‘\(p\)’ for any given \(SK\) in the form

\(p = A + Bs \sin{(SK)}\) (12)

To determine \(A\) and \(B\) we choose, for example \(SK = 30°\) and 165°. By solving the linear equations, we obtained \(A = 240\)°.372 and \(B = -15\)°.7429.

When \(SK = 0°\) or 180° equation (12) becomes \(p = A\). Hence we can determine the śīghra peri-
pheries of planets from the range of $SK = 0^\circ$ to $180^\circ$ which are listed in Table 12.

The above values of $\text{sighra}$ pherries are compared with other texts to draw a conclusion on our method of computation (see Table 13).

### 5 CONCLUDING REMARKS

In the above sections we have analyzed the discrete mandārīkas and $\text{sighrārīkas}$ given in the Grahalāghava of Gaṇeṣa Daivajña. We have obtained the ranges of the corresponding $\text{manda}$ pherries for all bodies and $\text{sighra}$ pherries for the five planets and compared them with those of the Āryabhaṭiya and in the Sūryasiddhānta. We find that the ranges of $\text{manda}$ pherries of the Sun and the Moon vary as in the Sūryasiddhānta. However the results obtained are approximate ones; the reasons for this are:

1. The equation of the centre and the conjunction ($\text{manda}$ and $\text{sighraphalas}$) given in the GL are over wide intervals of $15^\circ$; and
2. The given numerical values are in integers, avoiding fractions in the case of the five planets.

The constants $A$ and $B$ in equations (7) and (8) obtained are slightly different for different choices of related linear equations. This discrepancy is due to the approximations mentioned above.

### 6 NOTES

1. Āryabhaṭa I (born 476 C.E) gives, just in one śloka (verse), the rule to obtain the $jyā$ (R sine) of any angle between $0^\circ$ to $90^\circ$ at an interval of $3^\circ$ $45'$. He gives the differences between successive values in arc-minutes ($\text{kālās}$). Āryabhaṭa's value for the constant co-efficient $R$ is $3438'$, which is the nearest integer value to a radian.

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